STABILITY ANALYSIS OF LAMINATED COMPOSITE CIRCULAR PLATES WITH HOLES

A thesis submitted in partial fulfillment of the requirements for the degree of

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In
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By

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CERTIFICATE

This is to certify that the thesis entitled, **"STABILITY ANALYSIS OF LAMINATED COMPOSITE CIRCULAR PLATES WITH HOLES"** submitted by **RAJANI KANT SINGH** bearing roll no. **111CE0439** of C**ivil Engineering Department,** National Institute of Technology Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter presented in this thesis has not been submitted to any other University / Institute for the award of any other Degree or Diploma.

Date: 10th May, 2015

Place: Rourkela

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RAJANI KANT SINGH

ABSTRACT

A composite material is made up of two or more than two materials and possesses properties which are different from any of its constituent materials. It has advantages mainly because of its high stiffness to weight ratios and high strength to weight ratios. The mechanical properties of the laminated composite material can be varied by suitably changing the lamination scheme. In such materials, the fibres are the load carrier, and the matrix, having low modulus and high elongation, helps in providing the required flexibility and also helps in keeping the fibres intact.

Fiber-reinforced composites are usually thin plates. They are often subjected to compressive loads which when it reaches critical buckling load has a possibility of failure. Hence the buckling behavior of the composite plates has always been a major concern for the researchers. The computation of buckling load is important to predict the behaviour of structures under dynamic loads. During designing of structures subjected to compressive loading, knowledge of buckling characteristics of the comprising elements is necessary in order to prevent overloading of the structure

The present study mainly deals with the buckling characteristics of annular laminated composite plates. Circular plates with holes will be studied for their stability under buckling. The plates considered are thin plates. The mode shapes for different boundary conditions are to be obtained using finite element package, ANSYS 13.0. The effect of various parameters like fibre orientation, location of holes, boundary conditions, number of layers etc., on the buckling load will be studied. The effect of holes sizes as well as that of change in thickness of plates and various other parameters on the buckling load will also be studied.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

A composite material is made up of two or more than two materials and possesses properties which are different from any of its constituent materials. Laminated composite plates are made up of lamina bonded together and made up of materials chemically different from each other but combined macroscopically. A layer of composite material is defined as a lamina and stacking laminae forms a laminated composite plate.

Laminated composites are being increasingly used in the aerospace and automobile industries and also in civil engineering structures as an option to steel, wood, and concrete. The fiber reinforcement provides most of the strength and acts as the main load-carrying member. The fiber reinforcement can be continuous, discontinuous, or particles. Although various materials can be used as fiber reinforcement in FRP, the most commonly used are glass, carbon, and organic (aramid or Kevlar) fibers [3]. Glass fiber reinforcement is the fiber reinforcement used in the FRP that will be analyzed in this thesis, thus the term fiberglass reinforced plastic.

Fig 1.1: **Continuous Strand Mat [3]**

The solid matrix that surrounds the fibers holds the fibers in the desired location, protects the fibers from the environment, and transfers loads between fibers. Matrix materials can be metal, polymers, carbon, or ceramics. Thermosetting polymers (resins) are the most common material used as a matrix and are the least expensive.

Fig 1.2: Microscopic photograph of FRP [3]

Annular plates are extensively used nowadays as structural members in aircraft design. The buckling behaviour of such plates is much written about in literature. These holes may be windows or access holes or holes for hardware to pass through. Sometimes these holes are required in order to deduct the weight of the structure. In many other applications these structural components are also made up of laminated composite material to further reduce its weight. As discussed earlier, this is due to the durability, corrosion resistance and low density of composite structures.

1.2 Importance of present study

Laminated composites are being increasingly used in the aerospace and automobile industries and also in civil engineering structures as an alternative to steel, wood, and concrete. The fiber reinforcement provides most of the strength and acts as the main load-carrying member. The fiber reinforcement can be continuous, discontinuous, or particles. Fiber-reinforced composites are usually thin plates. They are often subjected to compressive loads which when it reaches critical buckling load has a possibility of failure. Hence the buckling behavior of the composite plates has always been a major concern for the researchers. The computation of buckling load is important to predict the behaviour of structures under dynamic loads. During designing of structures subjected to compressive loading, knowledge of buckling characteristics of the comprising elements is necessary in order to prevent overloading of the structure.

1.2 Outline of present work

The present study mainly deals with the buckling characteristics of annular laminated composite plates. The effects of radius ratios, number of layers, boundary condition, and orientation of fiber on buckling characteristics of annular laminated plates were examined. The influence of lamination sequence of graphite epoxy and carbon epoxy composite laminates on the buckling behaviour was also studied.

This thesis contains five chapters. The first chapter presents a brief introduction on the importance and application of the present study.

In Chapter 2, review of the literature for the various studies conducted on the present topic has been enlisted. It also includes the methods that the authors adopted to get the results. This chapter also includes the objective and scope of the present study.

In Chapter 3, the finite element formulation for the solution of buckling problem has been presented. In this section the non-dimensional buckling load parameter is related to flexural rigidity based on the equations of equilibrium and using the laminate constitutive equations.

In Chapter 4, the procedure for modeling composite laminates using finite element package ANSYS 13.0, has been described stepwise.

In Chapter 5, the results deduced from the present investigation is compared with the papers and further analysis of the problem is done by changing the parameters such as variation in r_i/r_o ratios, aspect ratio, boundary condition, etc. and hence generalizing the results.

In Chapter 6, the conclusion drawn on the above studies have been described with a brief description.

CHAPTER 2

REVIEW OF LITERATURE

2.1 Introduction

Laminated composites are used extensively as thin plates. The load carrying capacity of such plates against buckling has therefore been extensively researched under various loading and boundary conditions. But very few research has been done on circular plates with cutouts. In order to check the stability of the various structural components such as those of civil and aerospace structures with inplane loading acting, buckling analysis is required. The literatures related with the topic were reviewed in order to get acquainted with the previous knowledge and theories adopted to conclude the results and hence emphasize the relevance of the present study.

2.2 Literature review

Yasui *et al* [1988] showed that the introduction of a hole into a plate does not always lead to the reduction of the buckling load and, in some cases; it may lead to increase in its buckling load.

Engelstad and Reddy [1992] predicted post-buckling response and failure of graphite/epoxy plates which may or may not have holes using stiffness reduction method in a FEA.

Lee and Hyer [1993] studied the failure mechanisms in a plate with a concentrically located circular hole and subjected to in-plane loads into the post-buckling range of deflections.

Britt [1994] from his study on panels with circular and elliptical cutouts concluded that for compression loading, panels with elliptical shaped cut-outs which have fiber orientation at certain angles to the lateral axis could have lesser buckling loads than panels with circular holes. This was true even though the circular cut-outs were made larger in area compared to the elliptical cut-outs.

Noor et al [1994] presented the buckling response characteristics of multilayered composite panels with cutouts, subjected to both mechanical and thermal loads simultaneously.

Hu and Lin [1995] analysed the buckling of symmetrically laminated composite plates subjected to uniaxial compression. Effect of different parameters like aspect ratios, thickness, central circular cutouts and boundary conditions on the buckling of plates were studied.

The effect of shape of holes on isotropic plates was investigated by Shakerley *et al* [1996]. Their study showed that small perforations should be placed away from the center of the plate and large perforations should be placed at the center of the plate in order to minimize the reduction in the elastic buckling load under uniaxial loading.

Ko [1998] studied compressive buckling of metal-matrix composite (MMC) plates with central square holes. He used the finite-element method and effects of hole size lamination schemes, boundary conditions, aspect ratio, etc on the compressive buckling strengths were studied. His analysis showed that for certain aspect ratio and boundary conditions the buckling strength increased with increase in hole size.

Lee et al [1998] analysed the buckling and post-buckling of delaminated circular composite plates.

Baltaci *et al* [2006] studied buckling of laminated composite circular plates with circular holes using the finite element method. The plates were subjected to uniform radial load. The effects of various parameters like hole sizes, hole locations, change in thickness and end conditions on the buckling load, and the different mode shapes were determined.

Baba [2007] conducted tests on laminated composite plates with circular and semicircular cutouts and with different boundary conditions. The obtained test results were compared with results obtained from finite element analysis. The result showed that plate orthotropy and boundary conditions were related to each other.

2.3 Objective of the present study

Composite materials are extensively used in many engineering applications including aerospace, pressure vessels, sports equipment, and automobile components. This is due to their high strength-to-weight and stiffness-to-weight ratios and controlled environments required for their manufacture. Cutouts are a part of structural components due to functional requirements, and to produce lighter and more efficient structures.

A critical review of the literature showed that there are many studies about static stability of laminated composite plates. However not much work has been done on the stability analysis of annular laminated composite plates. The effect of hole sizes and its location, boundary conditions, radius/thickness ratio, stacking sequence and ply orientation on the buckling load will be investigated by the use of the software ANSYS.

2.4 Scope of study

Circular plates with holes will be studied for their stability under buckling. The plates considered are thin plates. The mode shapes for different boundary conditions are to be obtained using finite element package, ANSYS 13.0. The effect of various parameters like fiber orientation, location of holes, boundary conditions, number of layers etc., on the buckling load will be studied. The effect of hole sizes as well as that of change in thickness of plates on the buckling load will also be studied.

CHAPTER 3

THEORITICAL FORMULATION

3.1 Introduction

The laminated composite circular plate provided with holes may be considered as an annular thin plate with inside radius, r_i, outside radius r_o, and hole radii r_d. The composite circular plate may have different end conditions at inside and outside edges. The circular plate which is clamped at the inside or the outside edge have the displacements and slopes at the nodes of the inside or the outside edges as zero. The boundary conditions which will be used in present investigation are:

C–F: Fixed at inner edge, free at outer edge;

F–C: Free at inner edge, Fixed at outer edge;

3.2 Equations of equilibrium

The equations of equilibrium for a circular plate in polar coordinates are [2]:

$$\frac{\partial N_{\theta}}{\partial r} + \frac{1}{r}\frac{\partial N_{r\theta}}{\partial \theta} = 0$$

$$\frac{\partial N_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial N_{\theta}}{\partial \theta} = 0$$

$$\frac{\partial^2 M_r}{\partial r^2} + \frac{2\partial^2 M_{r\theta}}{r\partial\theta\partial r} + \frac{1}{r^2}\frac{\partial^2 M_{\theta}}{\partial \theta^2} + q = 0$$

where N_r, N_θ, $N_{r\theta}$ are force resultants and M_r, M_θ and $M_{r\theta}$ are moment resultants.

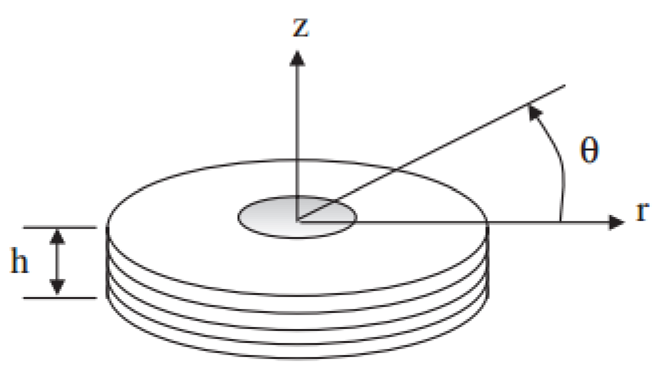


Fig 3.1: Circular plate with cutout and cylindrical coordinate system

3.3 Laminate Constitutive Equations

Strain-displacement relationships in cylindrical co-ordinates are as presented by [2]

$$\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \gamma_{\theta z} \\ \gamma_{rz} \\ \gamma_{r\theta} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_\circ}{\partial r} \\ \frac{1}{r}\frac{\partial u_\circ}{\partial \theta} \\ \frac{1}{r}\frac{\partial w_\circ}{\partial \theta} + \emptyset_r \\ \frac{\partial w_\circ}{\partial r} + \emptyset_r \\ \frac{1}{r}\frac{\partial u_\circ}{\partial \theta} + \frac{\partial v_\circ}{\partial r} \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial \emptyset_r}{\partial r} \\ \frac{1}{r}\frac{\partial \emptyset_\theta}{\partial \theta} \\ 0 \\ 0 \\ \frac{1}{r}\frac{\partial \emptyset_r}{\partial \theta} + \frac{\partial \emptyset_\theta}{\partial r} \end{Bmatrix}$$

Where u_o, v_o, w_o are mid-plane deflections in the r, $\boldsymbol{\theta}$ **and z directions** and ϕ_r, $\phi\boldsymbol{\theta}$ are rotations of transverse normal about r and $\boldsymbol{\theta}$ axes respectively.

The forces as well as the moment resultants (N, M) are related to the mid-plane strains and curvatures by the laminate constitutive equations where the force resultants are given by:

$$\begin{Bmatrix} N_r \\ N_\theta \\ N_{r\theta} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial u_\circ}{\partial \theta} \\ \frac{1}{r}\frac{\partial v_\circ}{\partial \theta} \\ \frac{1}{r}\frac{\partial u_\circ}{\partial \theta} + \frac{\partial v_\circ}{\partial r} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial \emptyset_r}{\partial r} \\ \frac{1}{r}\frac{\partial \emptyset_\theta}{\partial \theta} \\ \frac{1}{r}\frac{\partial \emptyset_r}{\partial \theta} + \frac{\partial \emptyset_\theta}{\partial r} \end{Bmatrix}$$

And the moment resultants are given as

$$\begin{Bmatrix} M_r \\ M_\theta \\ M_{r\theta} \end{Bmatrix} = \left[\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix}\right] \begin{Bmatrix} \frac{\partial u_\circ}{\partial \theta} \\ \frac{1}{r}\frac{\partial v_\circ}{\partial \theta} \\ \frac{1}{r}\frac{\partial u_\circ}{\partial \theta} + \frac{\partial v_\circ}{\partial r} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \frac{\partial \emptyset_r}{\partial r} \\ \frac{1}{r}\frac{\partial \emptyset_\theta}{\partial \theta} \\ \frac{1}{r}\frac{\partial \emptyset_r}{\partial \theta} + \frac{\partial \emptyset_\theta}{\partial r} \end{Bmatrix}$$

In short,

$$\begin{Bmatrix} N_i \\ M_i \\ Q_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} & 0 \\ B_{ij} & D_{ij} & 0 \\ 0 & 0 & S_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j \\ k_j \\ \gamma_m \end{Bmatrix}$$

where, A_{ij} is the extensional stiffness, D_{ij} denotes the bending stiffnesses, and B_{ij} the bending–extensional coupling stiffnesses.

$$\mathbf{A_{ij}} = \sum_{k=1}^{N} \mathbf{Q}_{ij}^{(k)}(\mathbf{h_{k+1}}\text{-}\mathbf{h_k}),$$

$$\mathbf{B_{ij}} = \frac{1}{2}\sum_{k=1}^{N} \mathbf{Q}_{ij}^{(k)}\left(\boldsymbol{h^2}_{k+1} - \boldsymbol{h^2}_{k}\right)$$

$$\mathbf{D_{ij}} = \frac{1}{3}\sum_{k=1}^{N} \mathbf{Q}_{ij}^{(k)}\left(\boldsymbol{h^3}_{k+1} - \boldsymbol{h^3}_{k}\right) \quad i, j = 1,2,6$$

Where $Q_{ij}^{(k)}$ denotes the k^{th} lamina material stiffness, with respect to the coordinates of the laminate and total number of layers in the laminate is denoted by N, and (h_k,h_{k+1}) being the thickness coordinates of the top and bottom of the k^{th} layer (Figure 3.2).

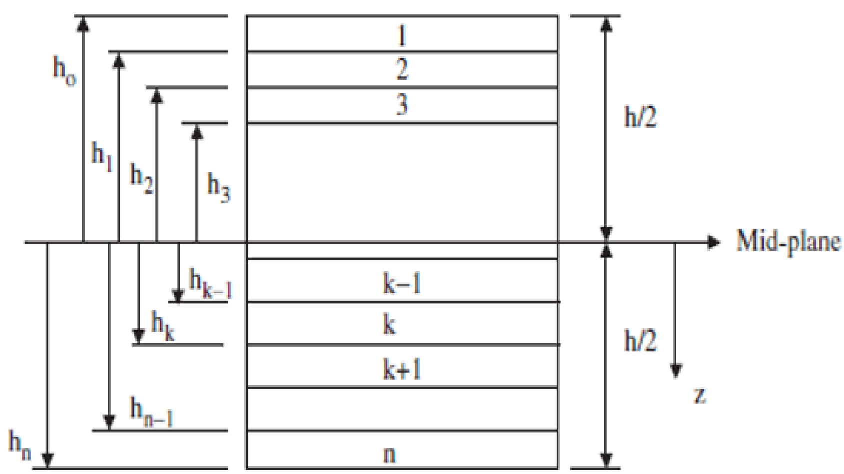


Fig 3.2: Locations of plies in a laminate

For problems concerning static stability, the equilibrium equations take the following form:

$$([K^e] - \lambda\,[S^e]) = 0$$

and the solution to this equation gives static buckling load parameter.

CHAPTER 4

MODELLING USING ANSYS 13.0

4.1 ANSYS Model

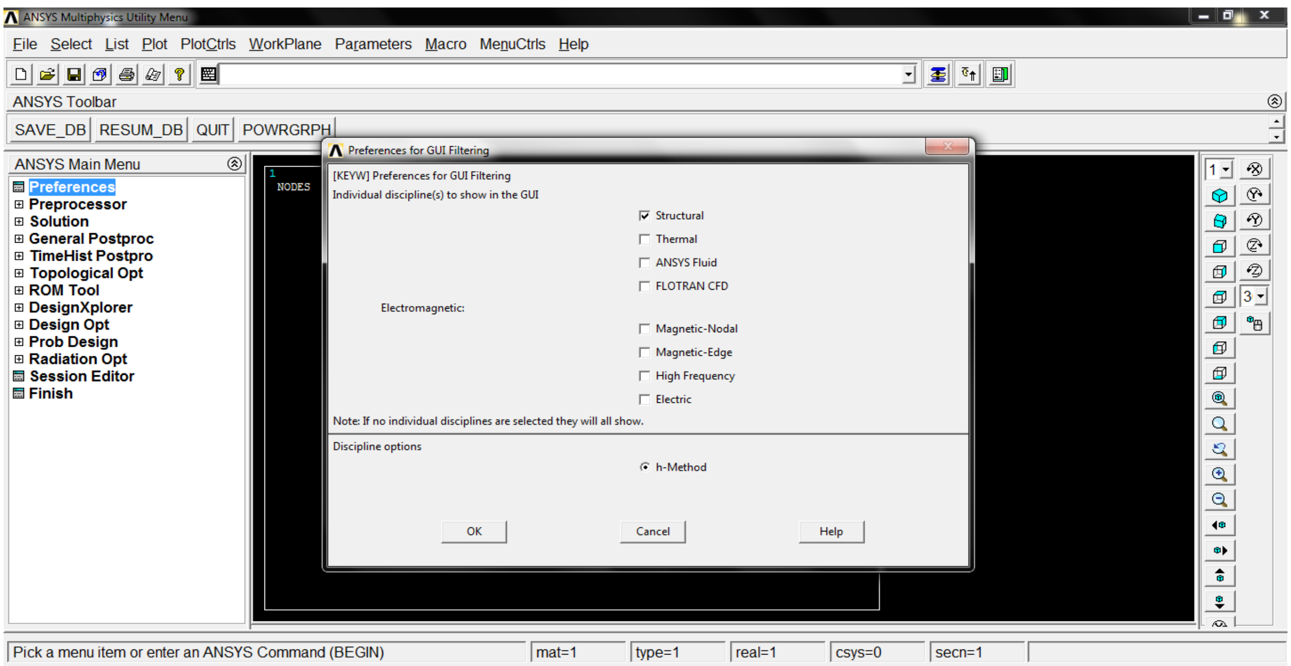
For modelling and buckling analysis of circular plates with holes, ANSYS 13.0 version was used which is a Finite Element Method (FEM) software.

The element type used is SHELL281 which is an 8 noded structural shell, suitable for analyzing thin to moderately thick shell structures. The element has 8 nodes with 6 degrees of freedom at each node. The accuracy in modeling composite shells is governed by the first order shear deformation theory.

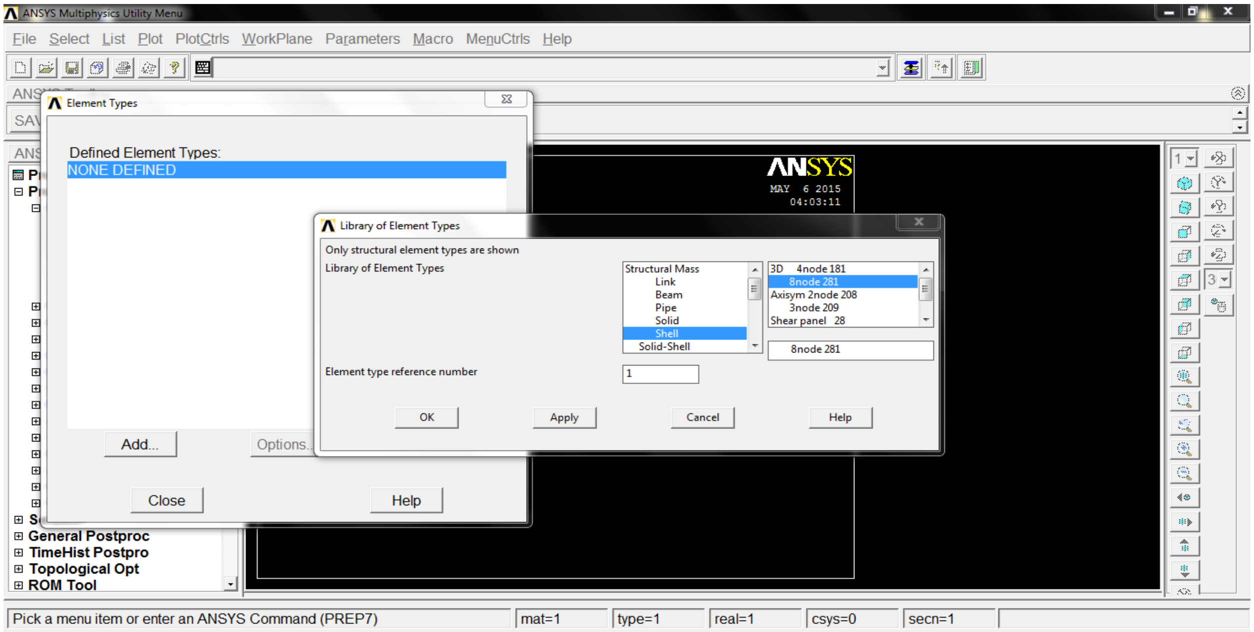
4.2 Procedural steps for modeling

The procedural steps required for modeling of laminated composite plates using ANSYS 13.0 version are as follows:

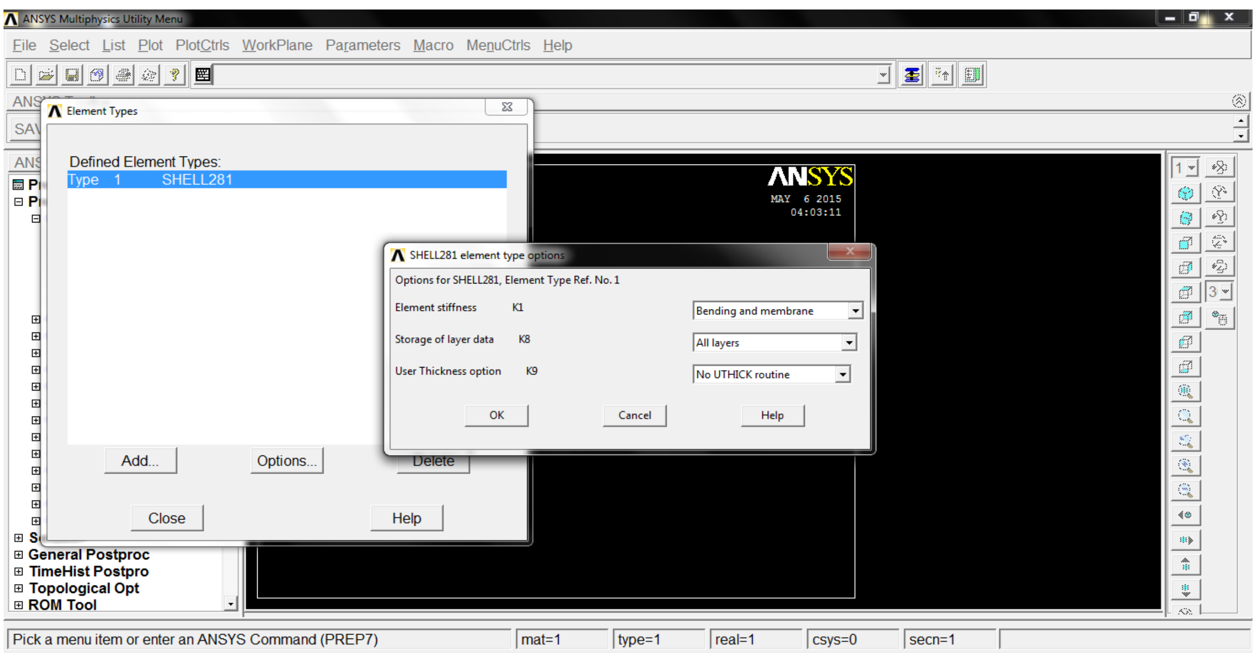
1. Preferences → Structural →Ok



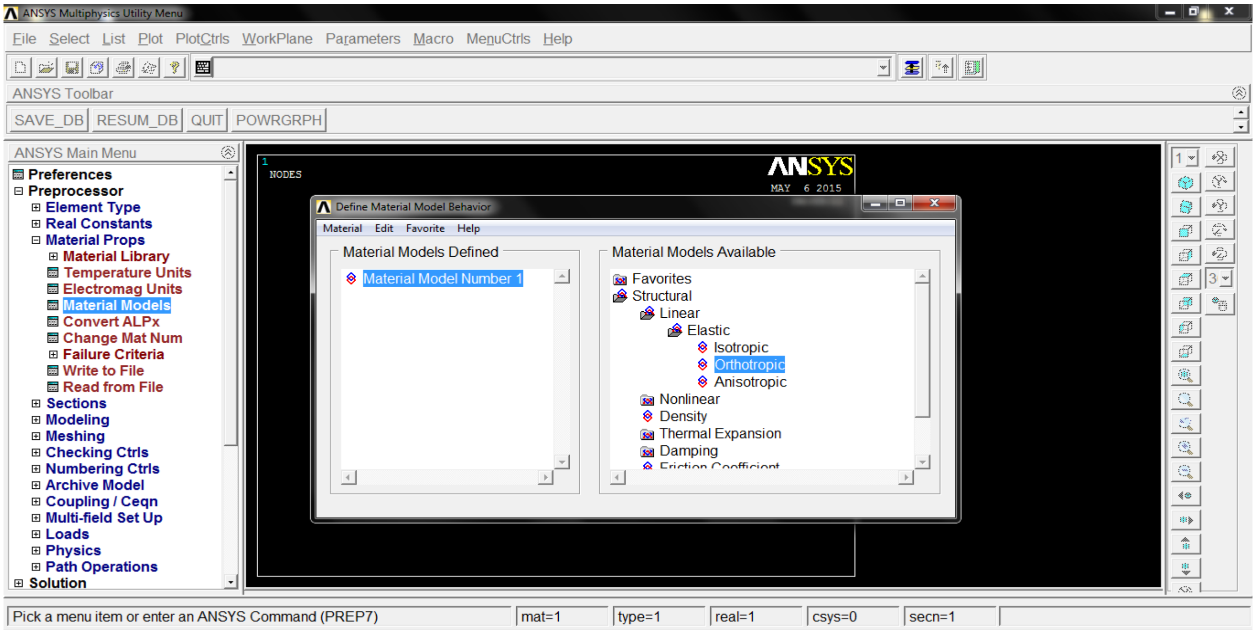
2. Preprocessor →Element type → Add → Shell → 8node 281 → Ok



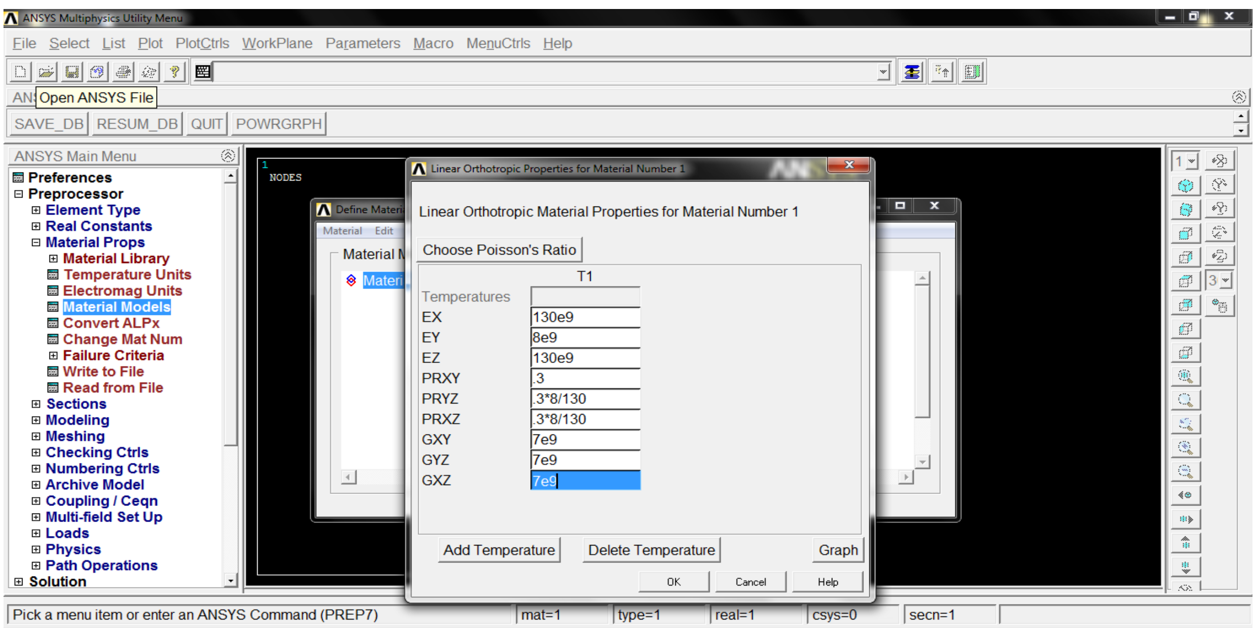
3. Select SHELL281 → Options → Storage of layer data – All layers → Ok



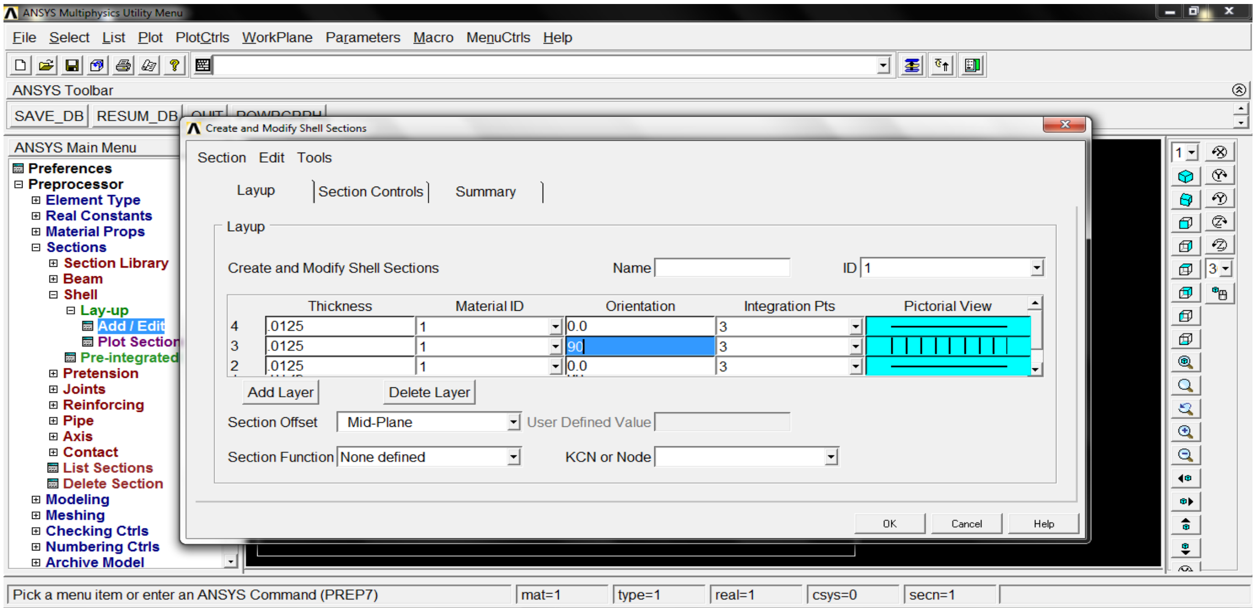
4. Preprocessor → Material Properties → Material Models → Structural → Linear → Elastic → Orthotropic



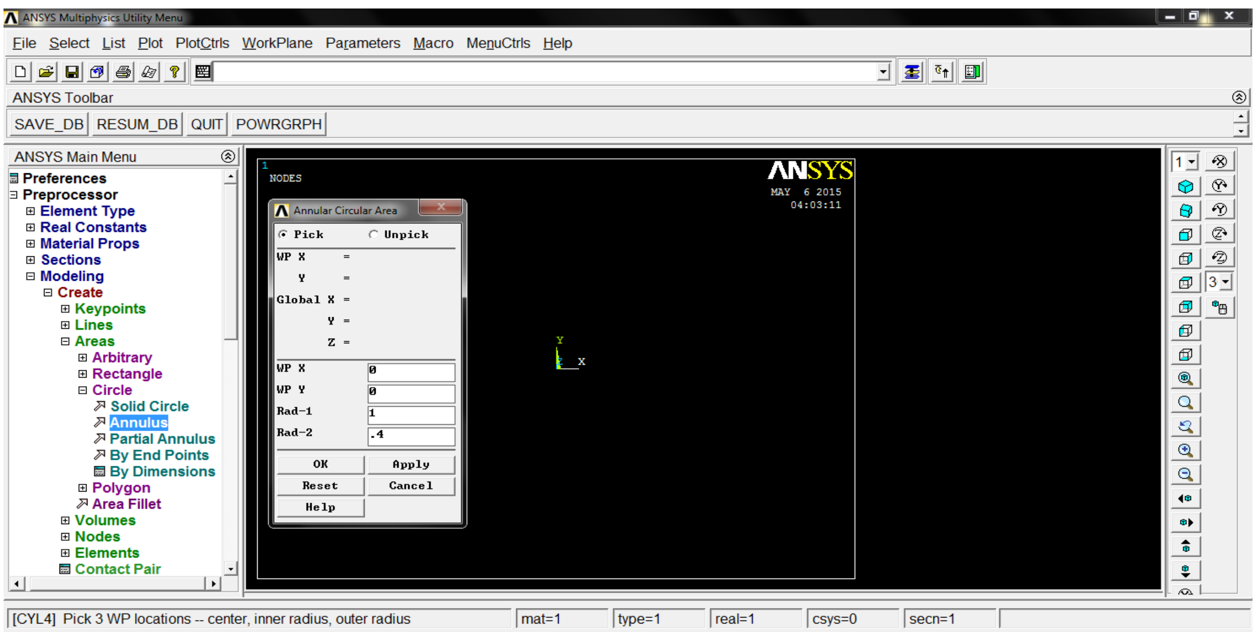
5. Here we enter the Linear Orthotropic Material Properties and then press Ok.



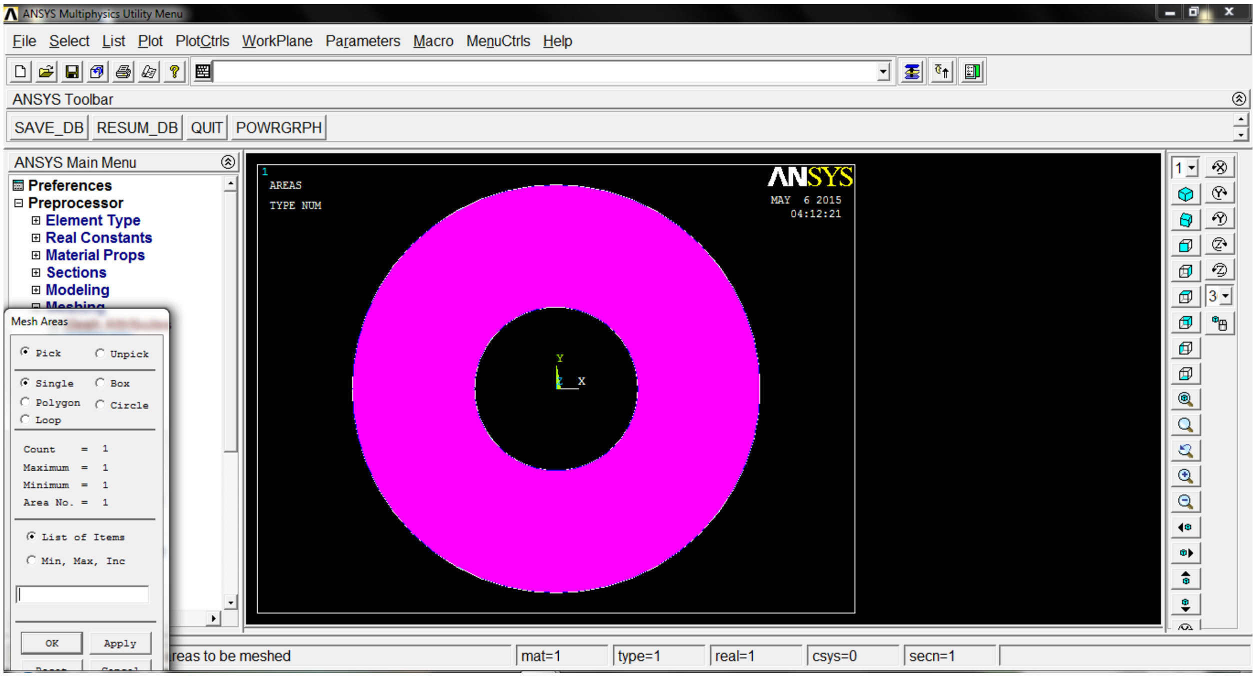
6. Preprocessor → Sections → Shell → Lay-up → Add/Edit → Enter thickness and orientation of each layer in the laminate by using Add Layer → Ok



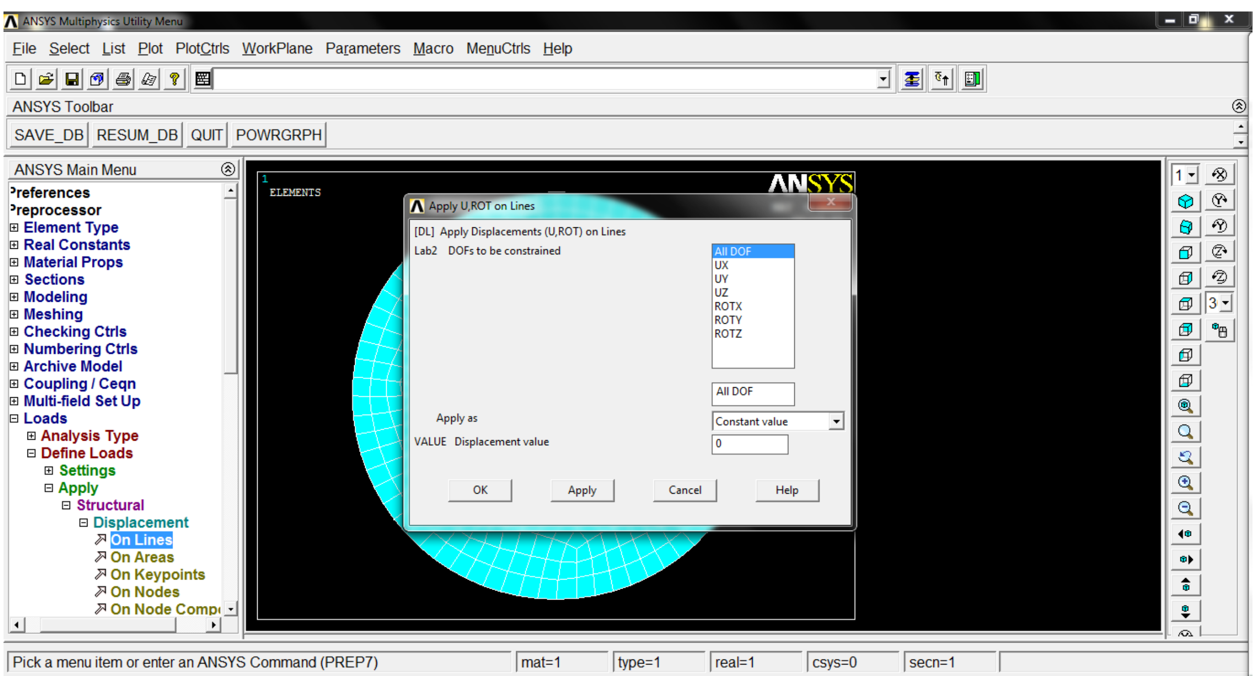
7. Preprocessor →Modelling → Create → Areas → Circle → Annular → Enter the dimensions of the required plates → Ok



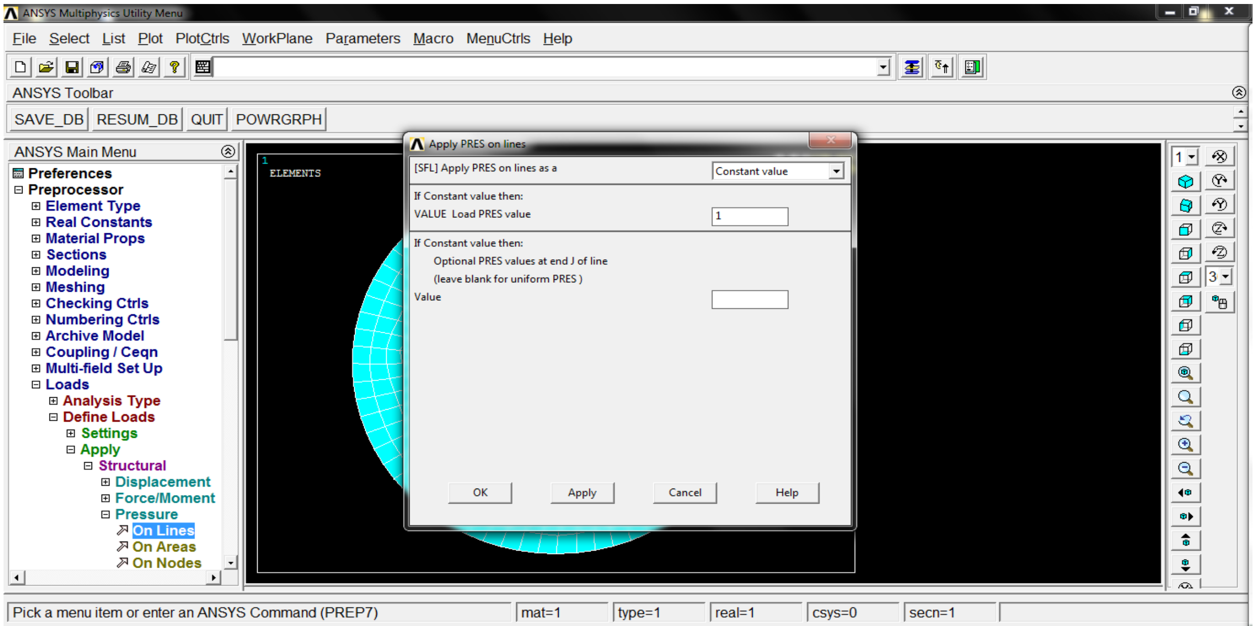
8. Preprocessor → Meshing → Mesh Tool → Size Controls: Global →Set → Mesh→ Enter the area to be meshed →Ok



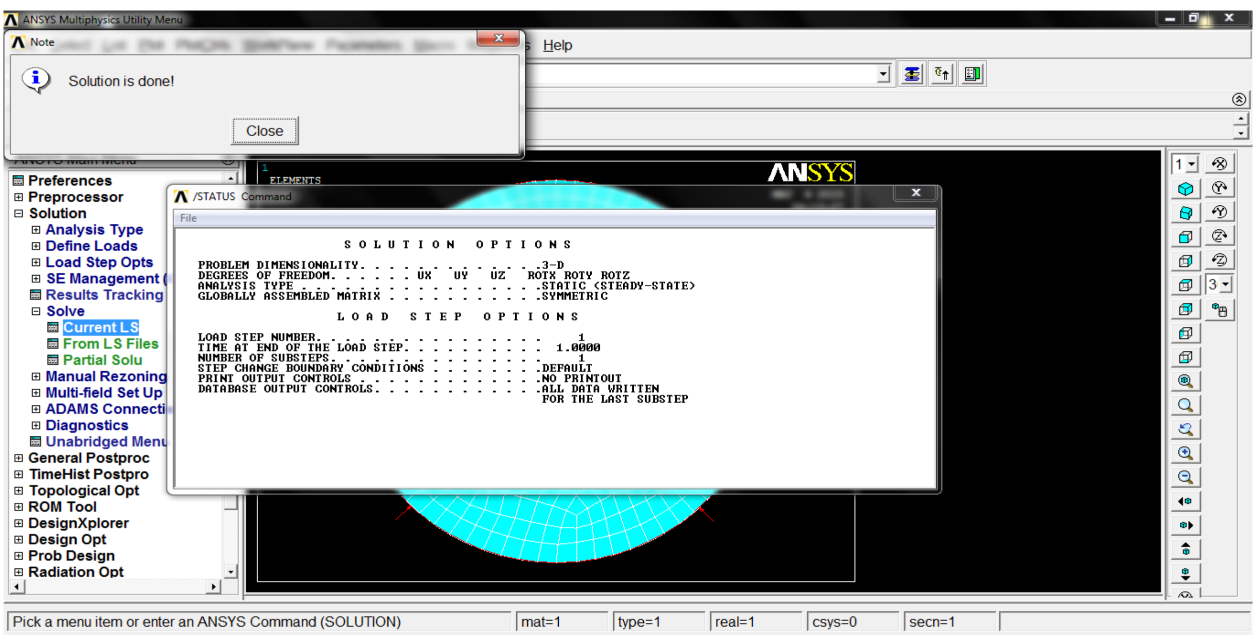
9. Preprocessor → Loads → Define Loads → Apply → Structural → Displacement → On Lines → Select the lines on which load is to be applied → Select the degree of freedom to be restrained → Ok



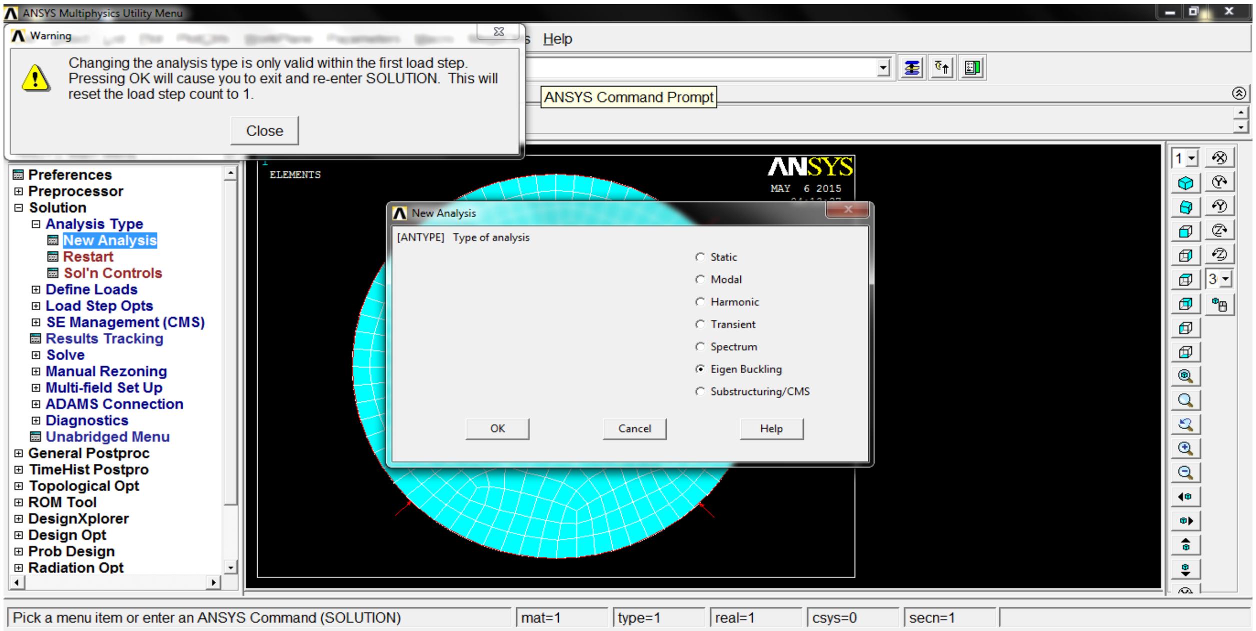
10. Preprocessor → Loads → Define Loads → Apply → Structural → Pressure → On Lines → Select the lines on which load is to be applied → Enter the pressure to be applied → Ok



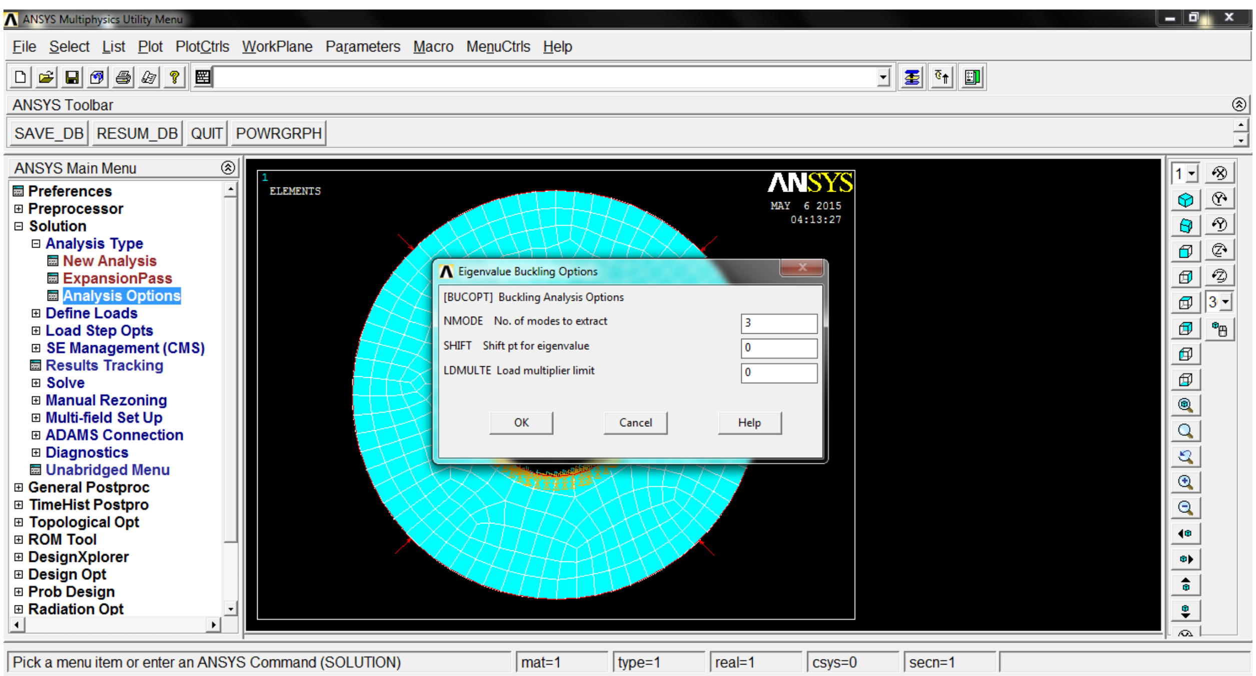
11. Solution → Solve → Current LS → Press ok in the Solve Current Load Step window



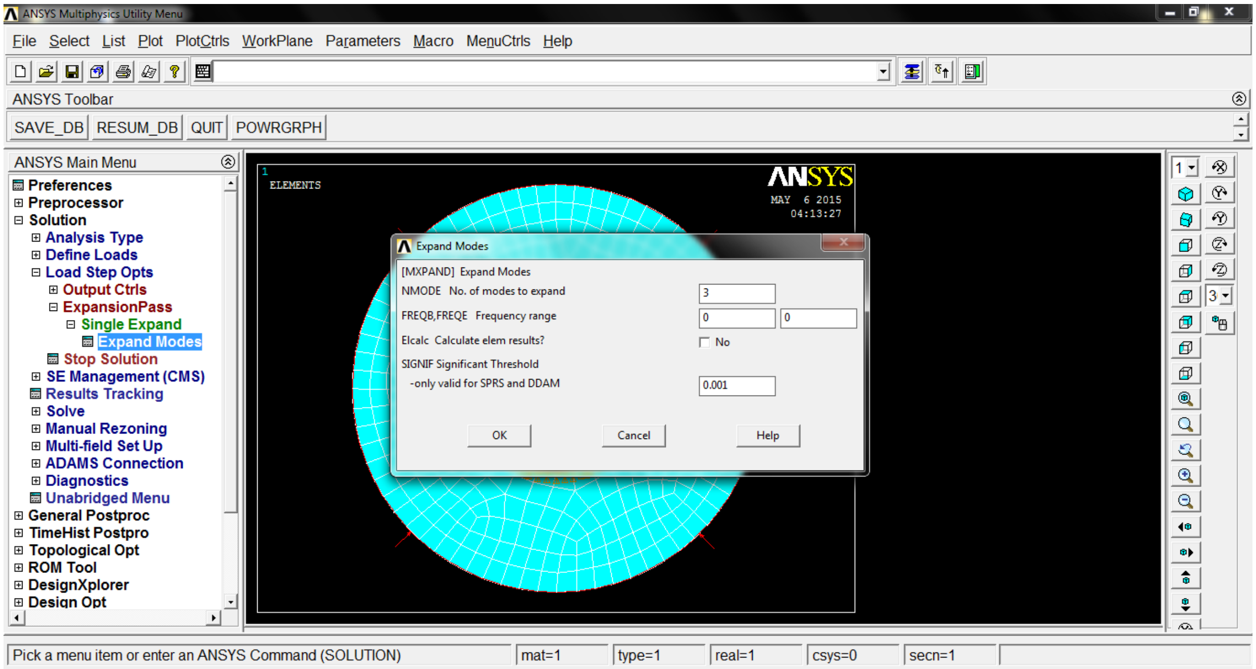
12. Solution → Loads → Analysis Type → New Analysis → Type of analysis – Eigen Buckling → Ok



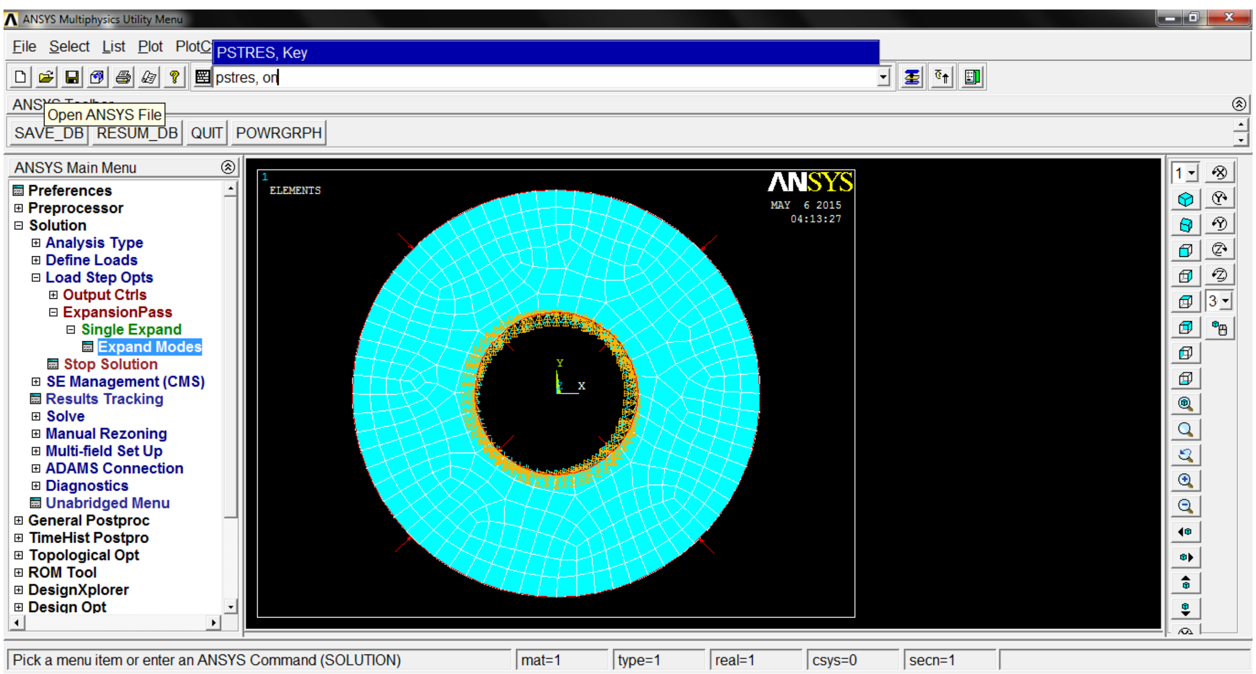
13. Solution → Loads → Analysis Type → Analysis Options → No of modes to extract (3) → Ok



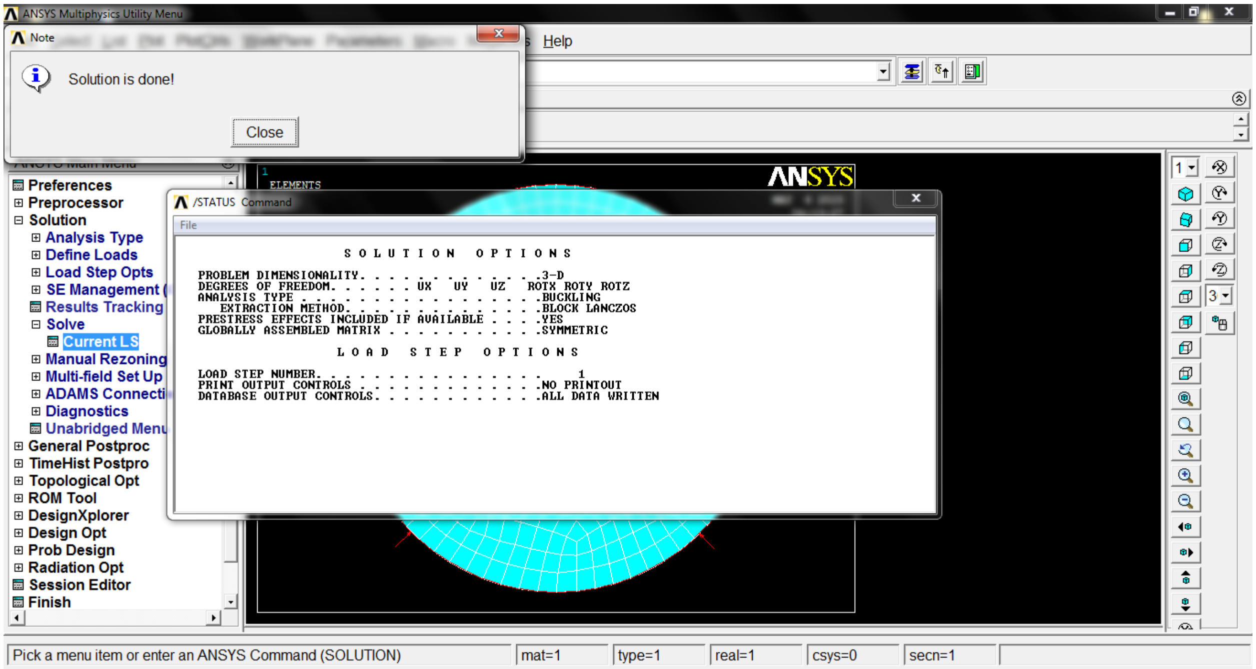
14. Solution → Loads → Load Step Options → Single Expand → Expand Modes → Enter no of Modes to Expand →Ok



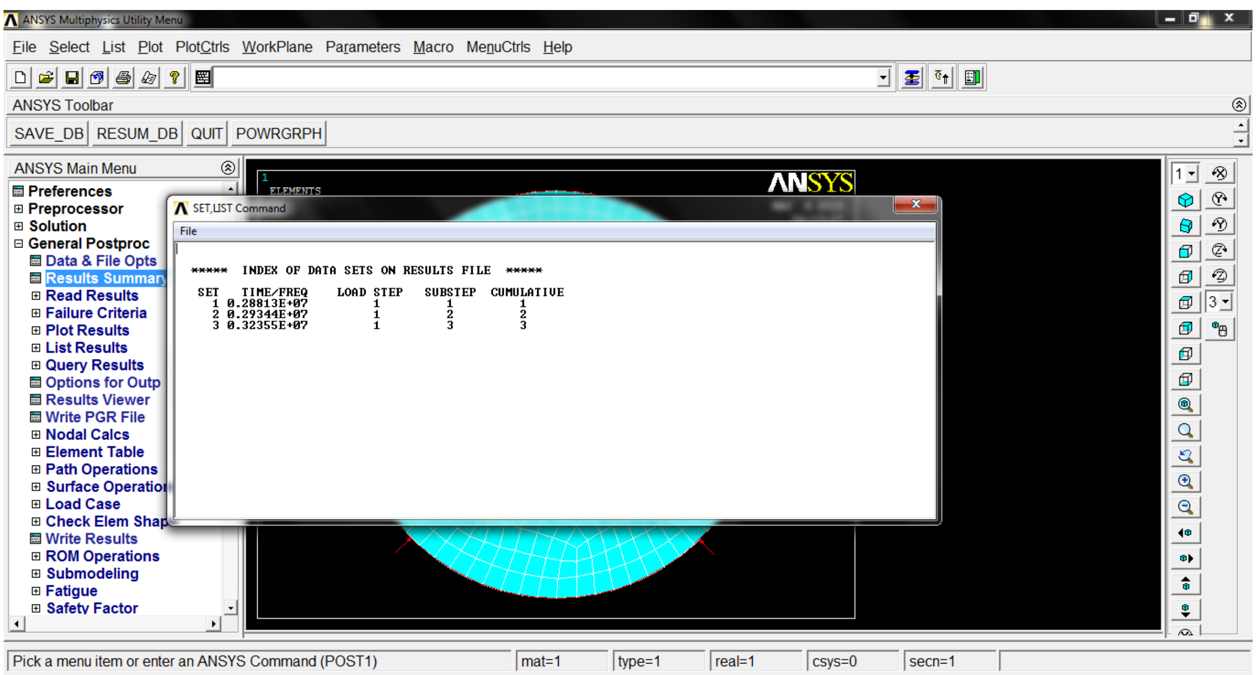
15. Now type PRESTRES, ON in the comment box and then press ENTER



16. Solution → Solve → Current LS → Press ok in the Solve Current Load Step window



17. General Postproc →Results Summary



CHAPTER 5

RESULTS AND DISCUSSION

5.1 Methodology

Two plates of different materials were analysed and the results were compared with the previous works done. The plates are

1) Isotropic circular plate with hole, made of steel
2) Orthotropic circular plate with hole, made of AS/3501 (Carbon epoxy plates)

Properties of plates

AS/3501 (Carbon Epoxy) Plates	**Steel plate**
Orthotropic Fiber orientation= $(90/0)_{2S}$ E_{XY} =E_{XZ}=138 GPa E_{YZ}= 8.96 GPa G= 7.1 GPa ν= 0.3 h = total thickness =0.1 m t = thickness of each layer= 0.0125m	Isotropic E = 210 GPa ν = 0.3 h = 0.0125 m

The boundary conditions were C-F ie. inner face was clamped and outer face was free for both cases.

5.2 Comparison of results

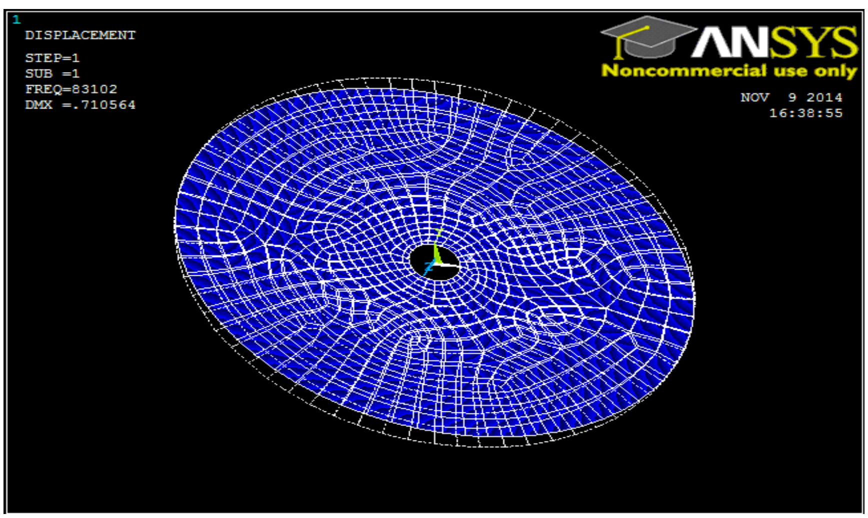
- ❖ A comparative study was done for buckling of circular steel plates with holes and the results by present formulation were compared with V. Mermertas *et al*. [18] and A. Baltaci *et al*. [2] and the results were found satisfactory as shown in Table 5.1. r_i/ r_o is the ratio of inside to outside radius.

K is non-dimensional buckling load parameter given by K = $\frac{N_r r_o^2}{E}$, where E is the flexural rigidity in the radial direction given by E = $\frac{E_R h_{max}^3}{12(1-\vartheta_{r\theta}\vartheta_{\theta r})}$

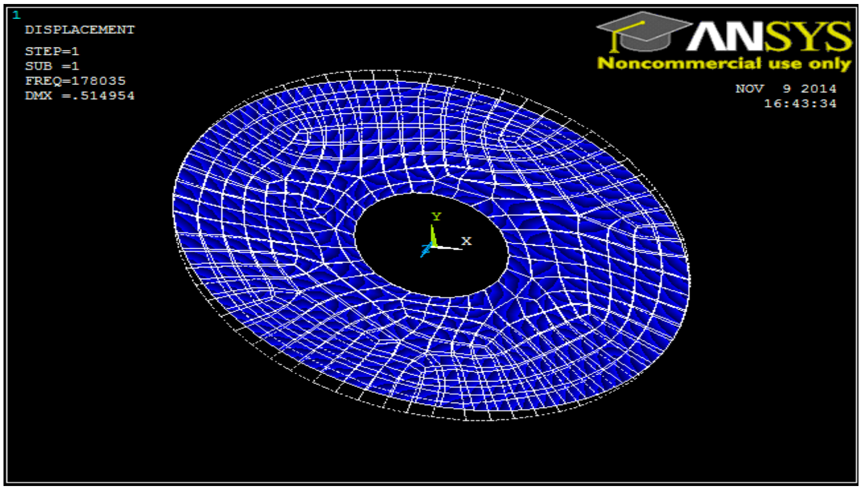
Table 5.1: Comparison of non-dimensional buckling load parameter of circular steel plates with holes

r_i/r_o	K		
	V. Mermertas [ ]	A. Baltaci [2]	Present
0.1	2	2.2	2.217
0.3	4.1	4.44	4.23
0.5	9	9.52	9.45

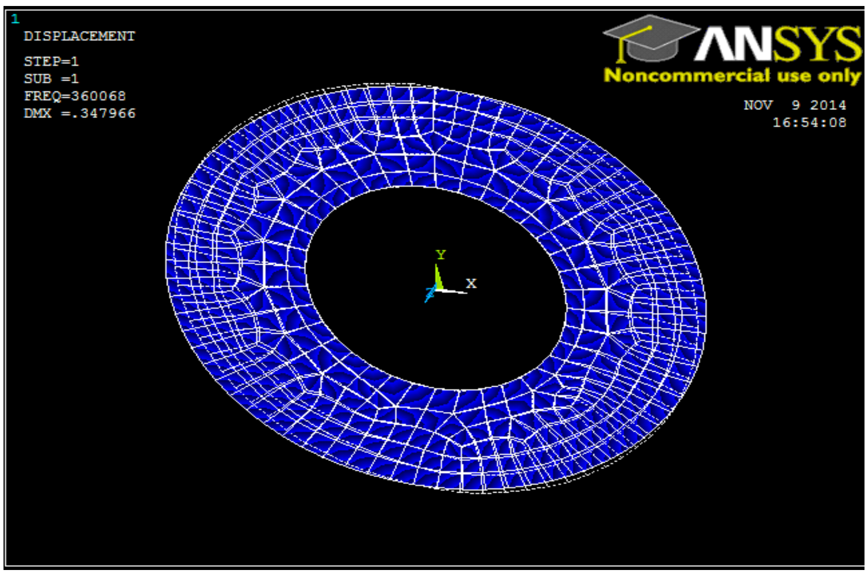
The deformed shape of the plate for first mode of buckling for various r_i/r_o ratios is shown in figure 5.1.



$r_i / r_o = 0.1$



$r_i / r_o = 0.3$



$r_i / r_o = 0.5$

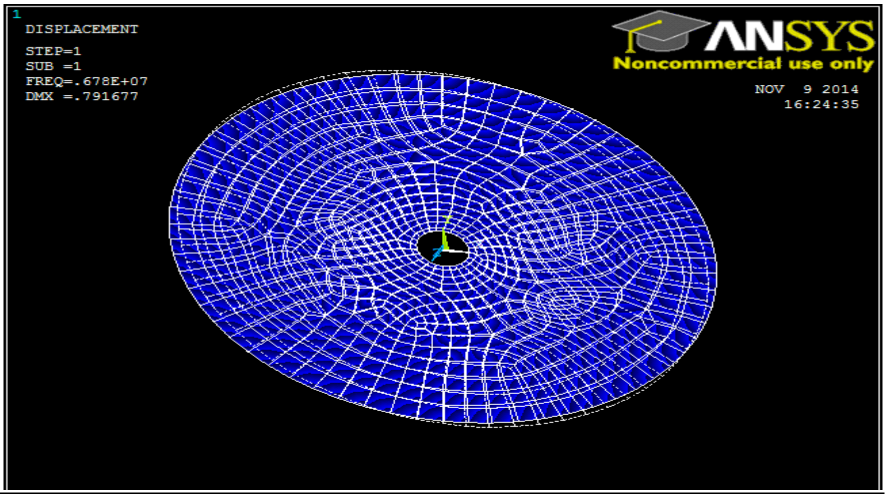
Figure 5.1: Deformed shape of steel plate for 1st mode for various r_i / r_o ratios

Next, a comparative study was done with orthotropic circular **AS/3501(Carbon Epoxy)** plates with holes. The results were compared with Baltaci *et al*. [2] as shown in Table 5.2 and it was found satisfactory. The inner face was clamped and outer face was free.

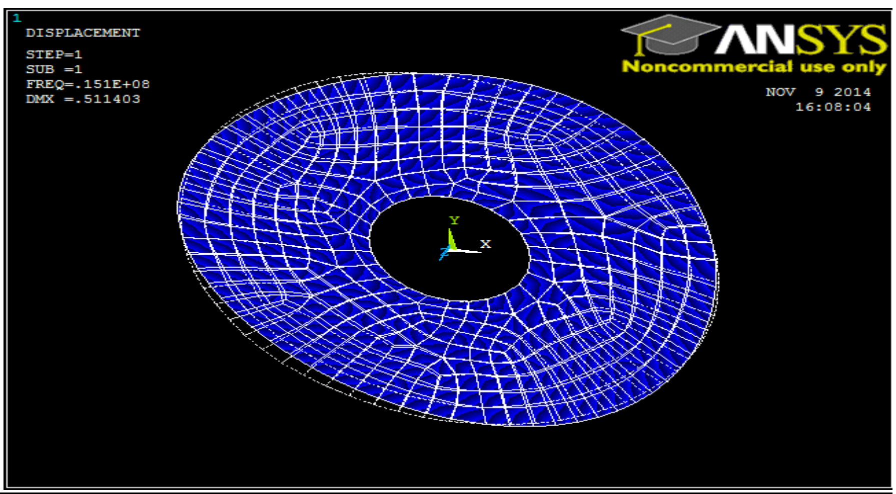
Table 5.2: Comparison of buckling load parameter of AS/3501(Carbon Epoxy) plates with holes

r_i/r_o	A. Baltaci [2]	Present
0.1	0.62	0.59
0.3	1.41	1.31
0.5	2.91	2.76
0.7	6.85	7.20

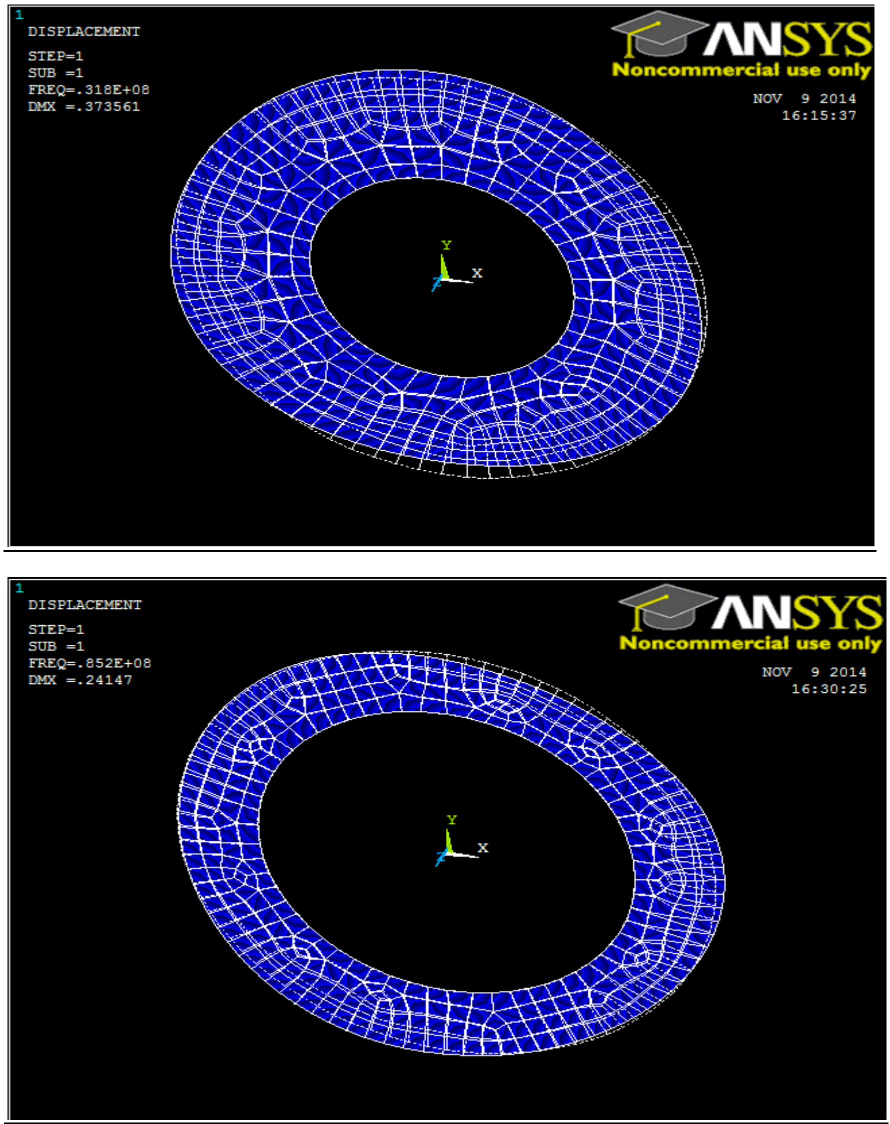
The deformed shape for first mode of buckling is given in figure 5.2 for various r_i/r_o ratios.



r_i /r_o =0.1



r_i /r_o =0.3



$r_i / r_o = 0.5$

$r_i / r_o = 0.7$

Figure 5.2: Deformed shape of AS/3501 plate for 1st mode for different r_i / r_o ratios

5.3 Analysis of the present problem

The analysis of the problem is done by varying the different parameters such as

- r_i/r_o ratios
- Boundary conditions
- Thickness of the plate
- Fibre orientation
- Changing the number of layers , etc

Properties of material:

(1) AS/3501 (Carbon Epoxy) Plates

Orthotropic circular plate

Fiber orientation= $(90/0)_{2S}$

E_{XY} =E_{XZ} =138 GPa

E_{YZ} = 8.96 GPa

G= 7.1 GPa

ν= 0.3

h = total thickness =0.1 m

t = thickness of each layer= 0.0125 m

(2) Graphite Epoxy Plates

Orthotropic circular plate

Fiber orientation= $(90/0)_{2S}$

E_{XY} =E_{XZ} =130 GPa

E_{YZ} = 8 GPa

G = 7 GPa

ν = 0.3

h = total thickness =0.1 m

t = thickness of each layer= 0.0125 m

- The effect of various r_i/r_o ratios on the buckling load P of Clamped Free boundary condition of plate (1) with constant thickness and $(90/0)_{2s}$ fiber orientation of laminated fibres is shown in Table 5.3 and figure 5.3.

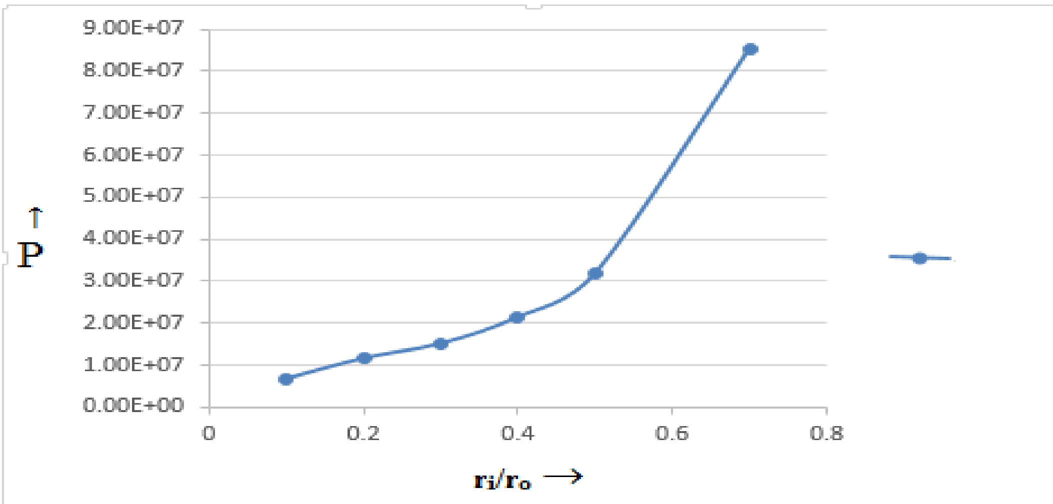
Table 5.3: Effect of various r_i/r_o ratios on P for (a) AS-3501 plate and (b) Graphite epoxy plates

r_i/r_o	0.1	0.2	0.3	0.4	0.5	0.7
P	0.6783e7	0.11645e8	0.15126e8	0.21433e8	0.3180e8	0.85237e8

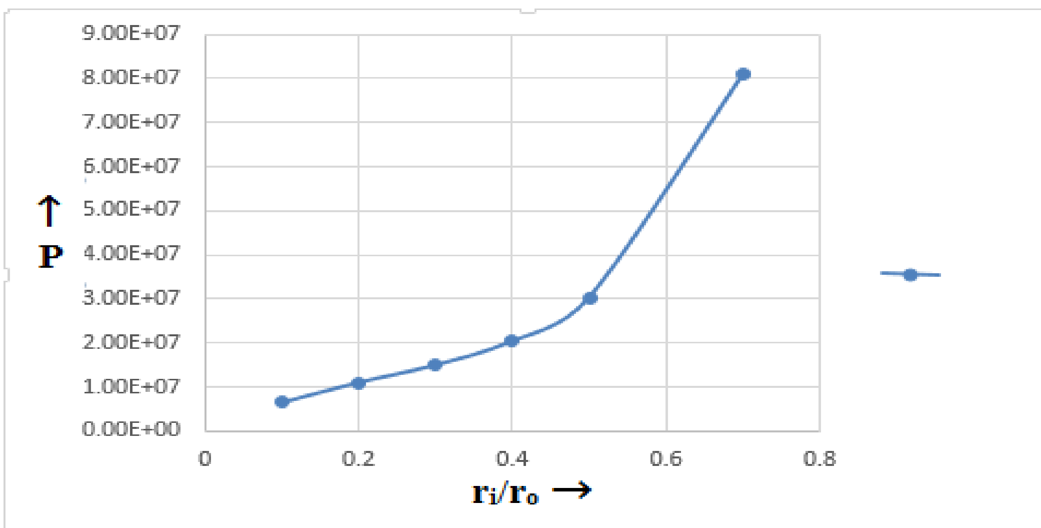
(a)

r_i/r_o	0.1	0.2	0.3	0.4	0.5	0.7
P	0.64404e7	0.11021e8	0.14974e8	0.20422e8	0.30218e8	0.80852e8

(b)



(a)



(b)

Fig 5.3: Effect of various r_i/r_o ratios on the buckling load P for (a) AS-3501 plate and (b) Graphite epoxy plates

The buckling load is found to increase with increase in the r_i/r_o ratio. If the outside radius remains constant and the inside radius is increased, then the radial length reduces and the plate thus becomes more stiffer [2]. This is also presented graphically in figure 5.3.

- The effect of change in boundary conditions on the buckling load of composite annular plate having constant thickness and $(90/0)_{2s}$ lamination scheme is shown in Table 5.4. C-F means inside edge fixed and outside edge free and F-C means inside edge free and outside edge fixed. As can be observed, the plate with inner edge free and outer edge clamped shows higher buckling load than the other plate. This is also seen in figure 5.4.

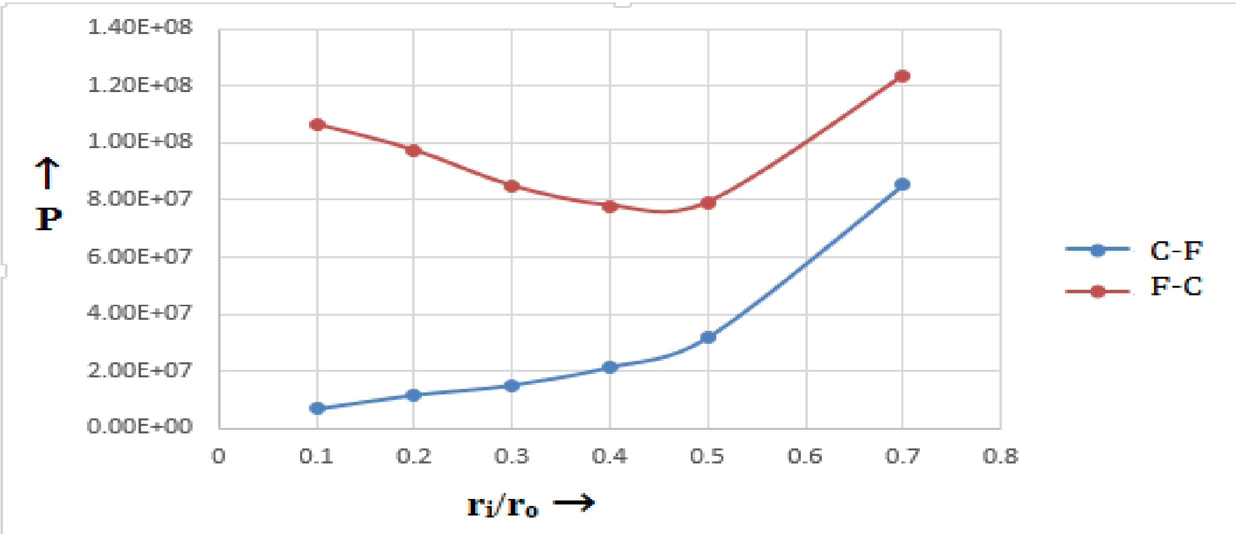
Table 5.4: Effect of change in boundary conditions on the buckling load P for (a) AS-3501 plate and (b) Graphite epoxy plates

r_i/r_o	0.1	0.2	0.3	0.4	0.5	0.7
P(C- F)	0.6783e7	0.11645e8	0.15126e8	0.21433e8	0.3180e8	0.85237e8
P(F- C)	1.0653e8	0.9740e8	0.8486e8	0.77978e8	0.7913e8	0.12365e9

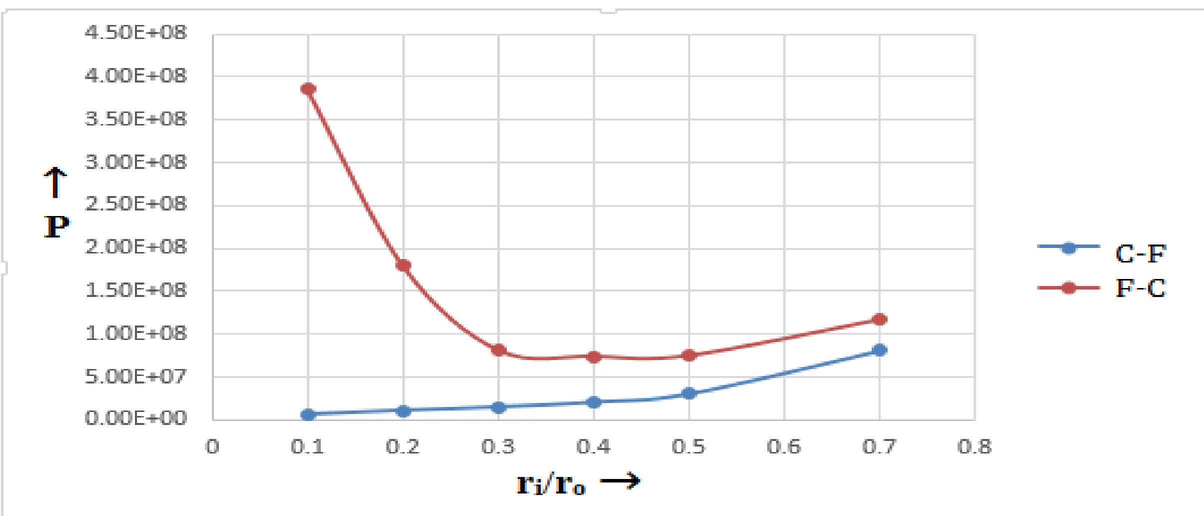
(a)

r_i/r_o	0.1	0.2	0.3	0.4	0.5	0.7
P(C- F)	0.64404e7	0.11021e8	0.14974e8	0.20422e8	0.30218e8	0.80852e8
P(F- C)	0.38616e9	0.17990e9	0.81007e8	0.74003e8	0.74973e8	0.11733e9

(b)



(a)



(b)

Fig 5.4: Effect of variations in boundary conditions on the buckling load P for (a) AS-3501 plate and (b) Graphite epoxy plates

- The effect of fibre orientation of composite plates on the buckling load of Plate (1) having uniform thickness and various r_i/r_o ratios is shown in Table 5.5.

Table 5.5: Effect of fibre orientation on the buckling load P for AS-3501 plate

r_i/r_o	0.1	0.2	0.3	0.4	0.5	0.7
P [$(90/0_2/90)_2$]	0.1747e8	0.2163e8	0.2927e8	0.37132e8	0.6501e8	1.3002e8
P [$(90/0)_4$]	0.14001e8	0.19087e8	0.24523e8	0.30885e8	0.47658e8	1.1475e8
P [$(+45/-45)_{2s}$]	0.96011e7	0.15269e8	0.20706e8	0.26721e8	0..41412e8	0.96589e8
P [$(90/0)_{2s}$]	0.67830e7	0.11645e8	0.15126e8	0.21433e8	0.3180e8	0.85237e8

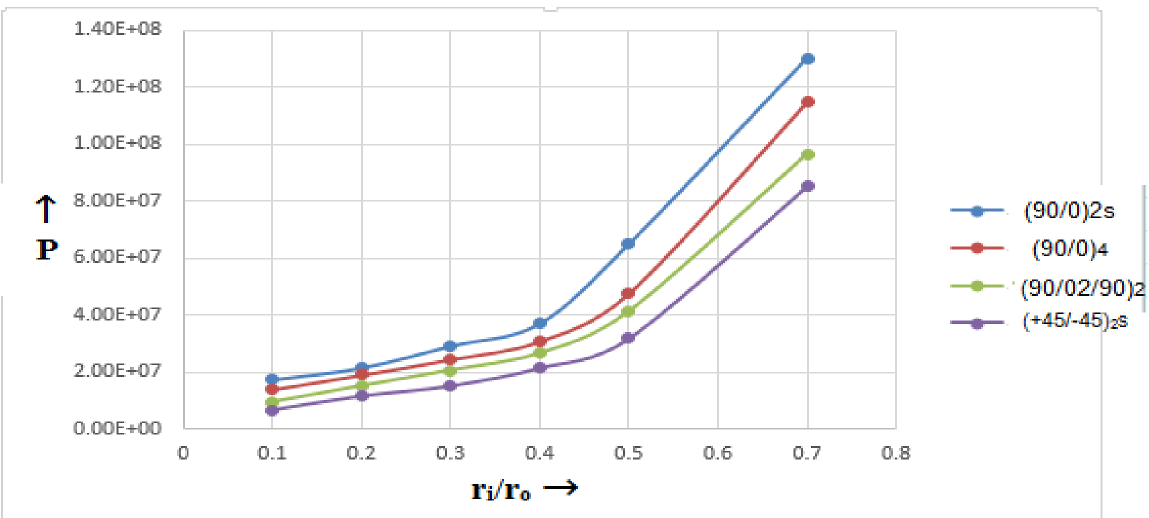


Fig 5.5: Effect of fibre orientation on the buckling load P for AS-3501 plate

From Table 5.5, it is seen that with increase in the number of layers, the buckling load increases. This behavior is also shown graphically in figure 5.5.

- The effect of the thickness variation on the buckling load of plate (1) with *$(90/0)_{2s}$* orientation of fiber was studied for varying thickness and is shown in Table 5.6 and figure 5.6.

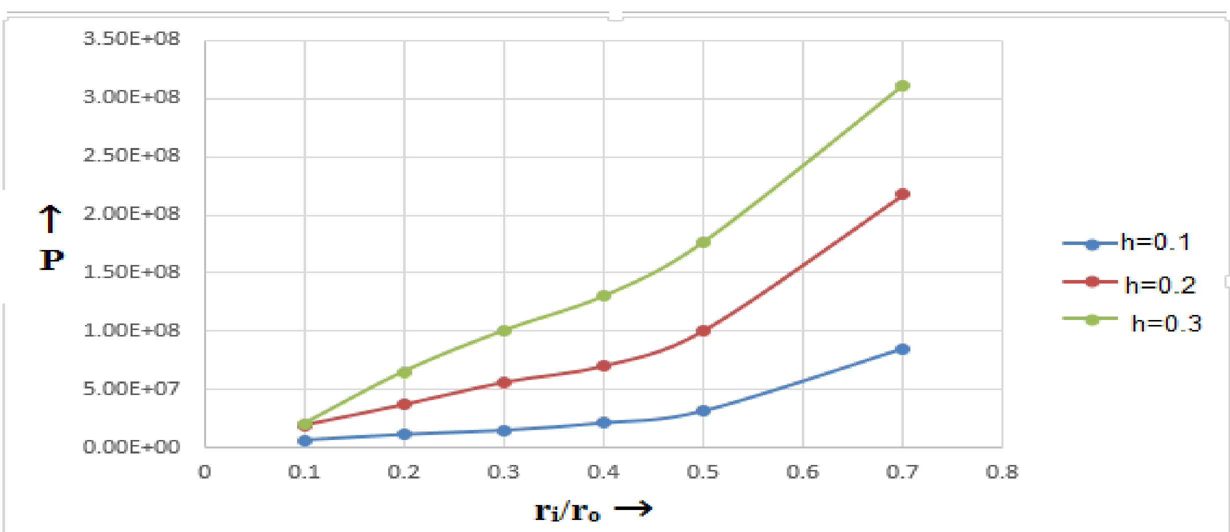
Table 5.6: Effect of change in thickness on the buckling load of clamped free (C-F) plates for (a) AS-3501 plate and (b) Graphite epoxy plates

r_i/r_o	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.7**
P (h=0.1)	0.6785e7	0.11645e8	0.15126e8	0.21433e8	0.31800e8	0.85237e8
P (h=0.2)	0.1955e8	0.37132e8	0.56132e8	0.70405e8	0.10072e9	0.21775e9
P (h=0.3)	0.2060e8	0.65363e8	0.10052e9	0.13001e9	0.17620e9	0.31102e9

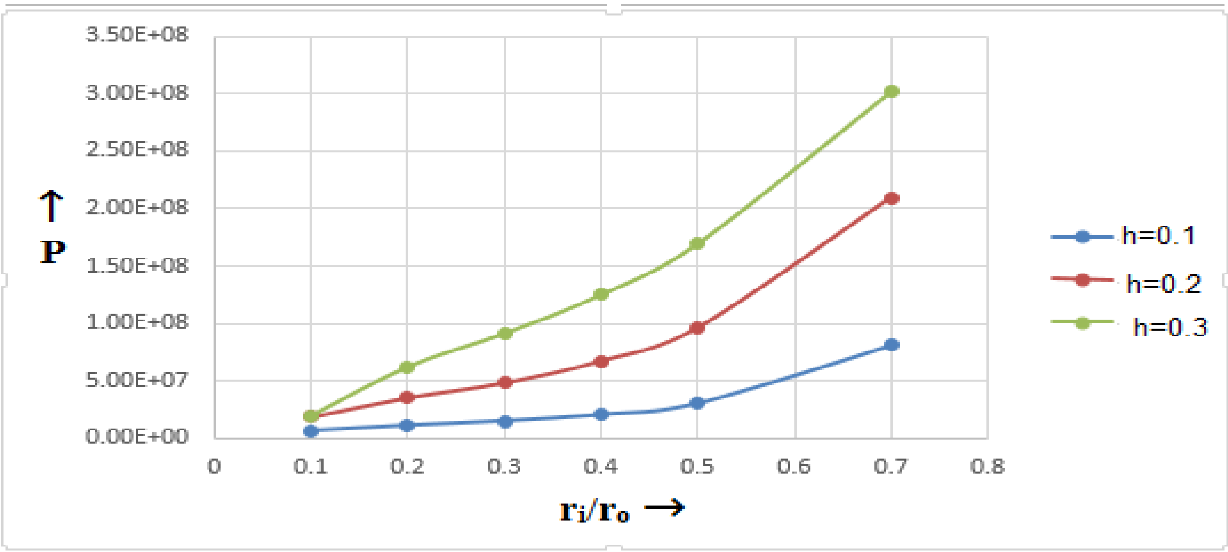
(a)

r_i/r_o	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.7**
P (h=0.1)	0.64404e7	0.11021e8	0.14474e8	0.20422e8	0.30218e8	0.80852e8
P (h=0.2)	0.18669e8	0.35313e8	0.48270e8	0.67436e8	0.96341e8	0.20934e9
P (h=0.3)	0.20097e8	0.62542e8	0.91376e8	0.12505e9	0.16948e9	0.30147e9

(b)



(a)



(b)

Fig 5.6: Effect of change in thickness on the buckling load of clamped free (C-F) plates for (a) AS-3501 plate and (b) Graphite epoxy plates

- Table 5.6 shows the effect of varying thickness on buckling load of clamped free plates. In both cases, the buckling load is found to increase with increasing thickness of the plate showing that the plate gets stiffer. However as r_i/r_o ratio increases, it is found to decrease and then increase in the second case. This behavior is also observed for free clamped plates as observed in Table 5.7 and figure 5.7.

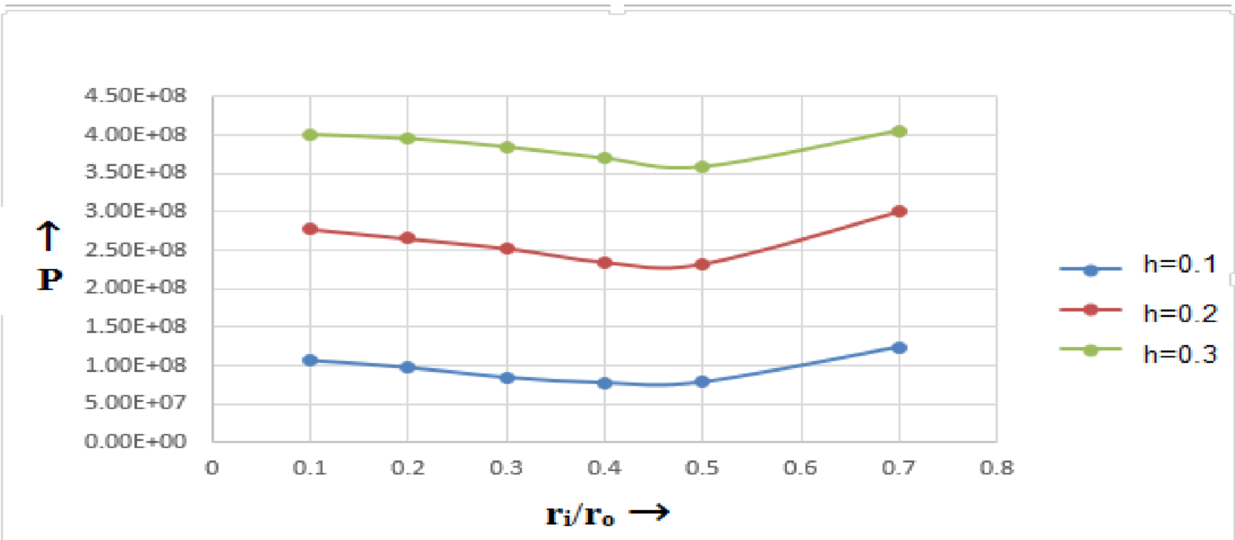
Table 5.7: Effect of change in thickness on the buckling load of free clamped (F-C) plates for (a) AS-3501 plate and (b) Graphite epoxy plates

r_i/r_o	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.7**
P (h=0.1)	0.1065e9	0.9740e8	0.8486e8	0.7798e8	0.7913e8	0.12365e9
P (h=0.2)	0.2777e9	0.2652e9	0.2528e9	0.2345e9	0.23226e9	0.30084e9
P (h=0.3)	0.40047e9	0.3952e9	0.3846e9	0.3702e9	0.3593e9	0.40540e9

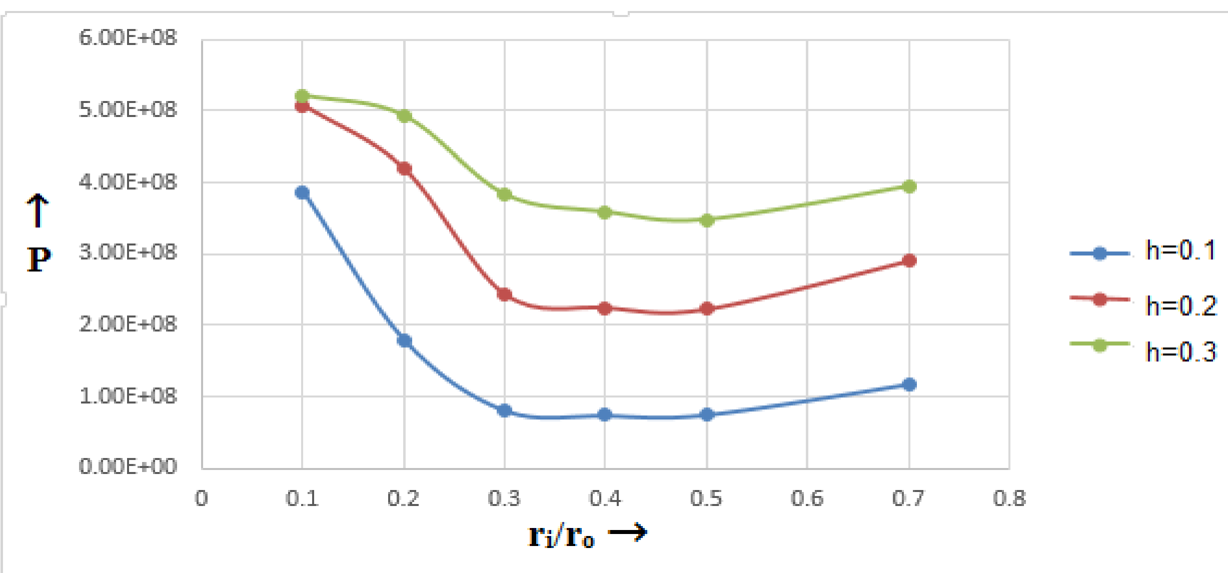
(a)

r_i/r_o	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.7**
P (h=0.1)	0.38616e9	0.17990e9	0.81007e8	0.74003e8	0.74973e8	0.11733e9
P (h=0.2)	0.50769e9	0.41967e9	0.24368e9	0.22481e9	0.22247e9	0.28971e9
P (h=0.3)	0.52127e9	0.49728e9	0.38272e9	0.35833e9	0.34759e9	0.39416e9

(b)



(a)



(b)

Fig 5.7: Effect of change in thickness on the buckling load of free clamped (F-C) plates for (a) AS-3501 plate and (b) Graphite epoxy plates

- The effect of change in the number of layers of annular laminated composite plates on buckling load for plate (1) with $(90/0)_n$ fiber orientation, where n =1, 2, 3 is shown in Table 5.8 and figure 5.8.

Table 5.8: Effect of change in number of layers on the buckling load P of free clamped (F-C) plates

r_i/r_o	0.1	0.3	0.5	0.7
P (4 no. of layers)	0.7045e7	0.1446e8	0.3355e8	0.9196e8
P (6 no. of layers)	0.10996e8	0.2024e8	0.3783e8	0.9994e8
P (8 no. of layers)	0.13996e8	0.2452e8	0.4766e8	1.1475e8

As seen from Table 5.8, with increase in number of layers, the buckling load also increases.

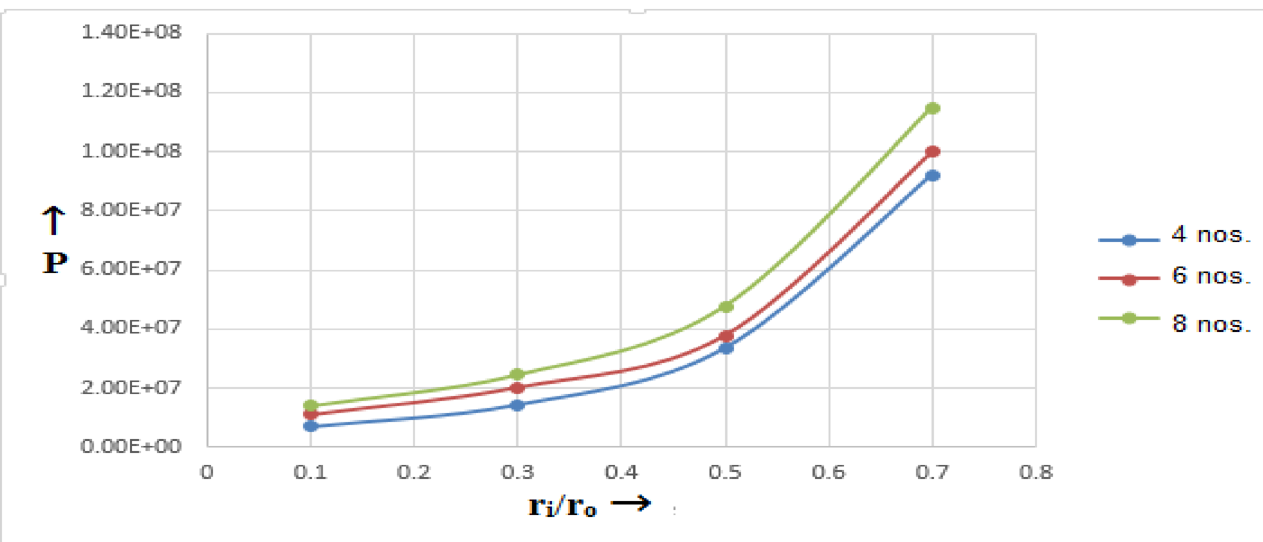


Fig 5.8: Effect of layer variations on the buckling load P of free clamped (F-C) plates

- The effect of location of holes on the buckling load P of Clamped Free composite circular plate having uniform thickness, $r_i/r_o = 0.1$, $r_h/r_o = 0.06$ and $(90/0)_{2s}$ orientation of laminated fibres is shown in Table 5.9 where r_h is the radius of the off center hole and r_{u1} is the distance of the off center holes from the center of the composite plate. As the distance of the hole from the centre of the plate increases, the buckling load increases.

Table 5.9: Effect of location of holes on the buckling load of the AS-3501 plate

r_{u1}/r_i	2.5	4.0	5.5	7.0	8.5	
P	0.2869e8	0.3204e8	0.3284e8	0.3343e8	0.34124e8	

(a)

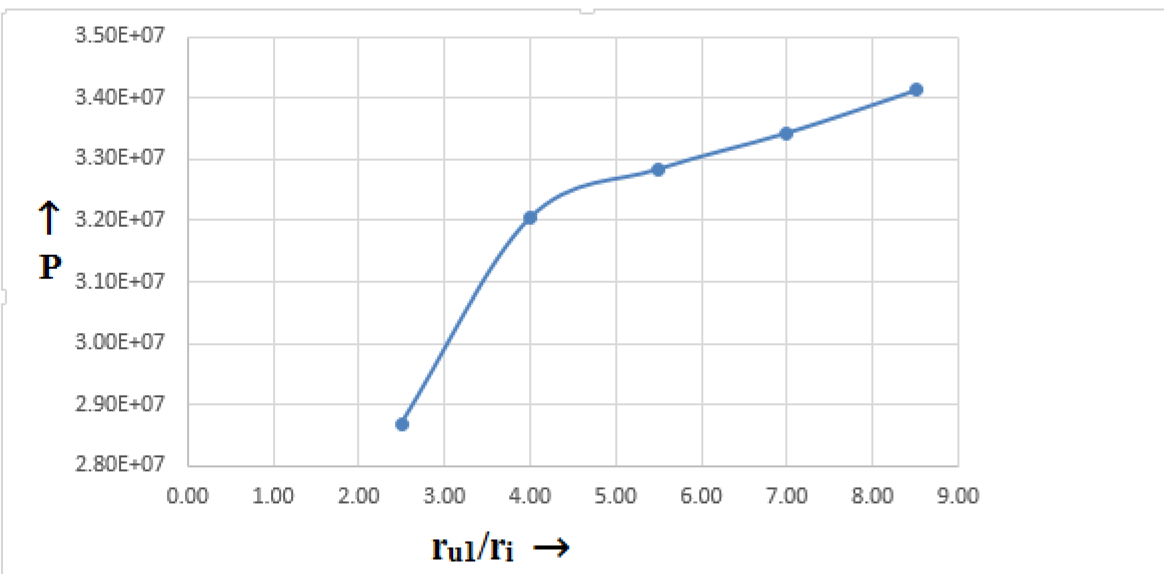


Fig 5.9: Effect of location of holes on the buckling load of the plates

- Table 5.10 shows the effect of size of cut-outs and its location on buckling load for clamped free end conditions. In these cases, the buckling load is found to decrease with increasing size of the hole but for a particular r_i/r_o ratio, the buckling load increases with increase in distance of the hole from the centre of the plate.

Table 5.10: Effect of size of hole and its location on the buckling load of the AS-3501 laminated composite plate

r_{u1}/r_i	**2.5**	**4.0**	**5.5**	**8.0**	**8.5**	
P (r_h/r_o =0.06)	0.2869e8	0.3204e8	0.3284e8	0.3343e8	0.34124e8	
P (r_h/r_o =0.08)	0.2441e8	0.30307e8	0.3227e8	0.3297e8	0.34009e8	
P (r_h/r_o =0.1)	0.1643e8	0.26721e8	0.3089e8	0.32505e8	0.33893e8	

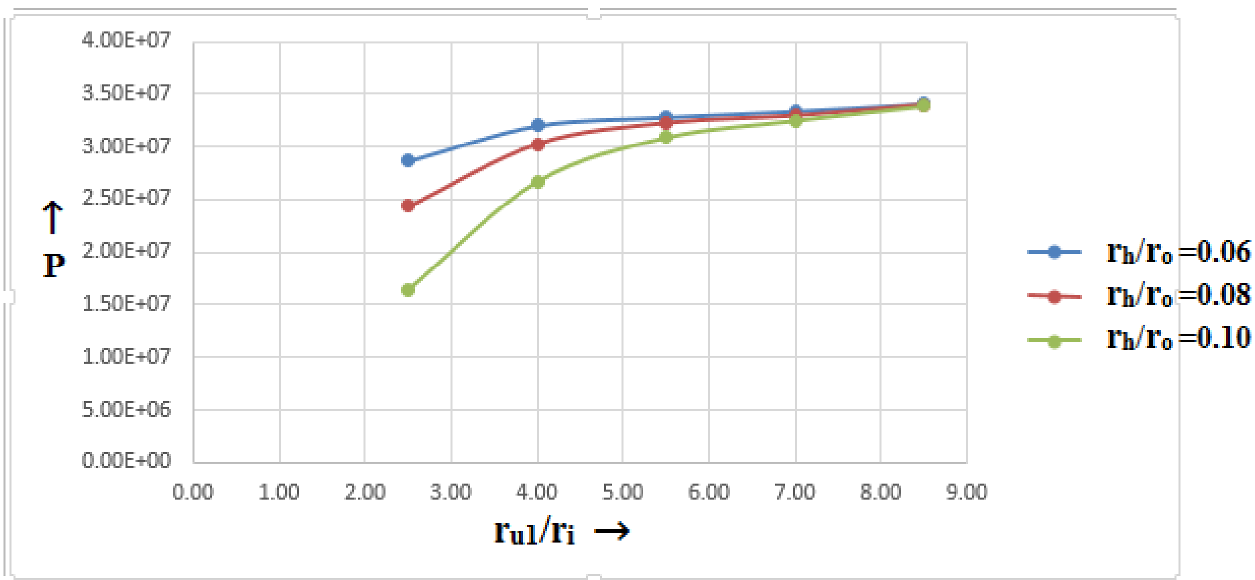


Fig 5.10: Effect of size of holes and its location on the buckling load of the laminated plate

➢ Table 5.11 shows the effect of location of two symmetric eccentric holes on the buckling load P of Clamped Free composite annular plate having uniform thickness, r_i/r_o = 0.1, r_h/r_o = 0.06 and $(90/0)_{2s}$ orientation of laminated fibres (AS-3501 plate)

Table 5.11: Effect of two symmetric eccentric holes on the laminated composite annular plates (AS-3501)

r_{u1}/r_i	2.5	4.0	5.5	7.0	8.5	
P	0.17728e8	0.2465e8	0.2616e8	0.2801e8	0.2956e8	

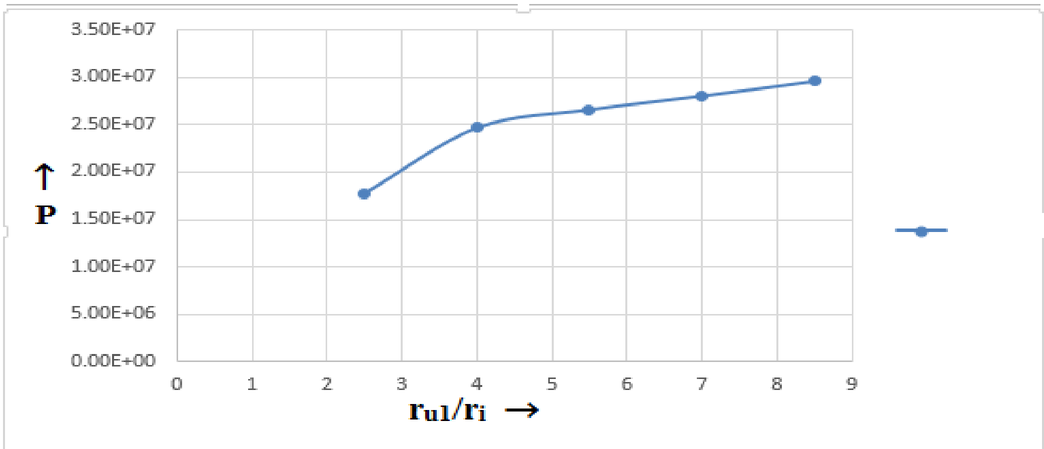


Fig 5.11: Effect of two symmetric eccentric holes on the laminated composite annular plates (AS-3501)

➢ Table 5.12 shows the effect of location of four symmetric eccentric holes on the buckling load P of Clamped Free composite annular plate having uniform thickness, r_i/r_o = 0.1, r_h/r_o = 0.06 and $(90/0)_{2s}$ orientation of laminated fibres (AS-3501 plate)

Table 5.12: Effect of four symmetric eccentric holes on the laminated composite annular plates (AS-3501)

r_{u1}/r_i	2.5	4.0	5.5	7.0	8.5	
P	0.11132e8	0.15261e8	0.17712e8	0.19275e8	0.20562e8	

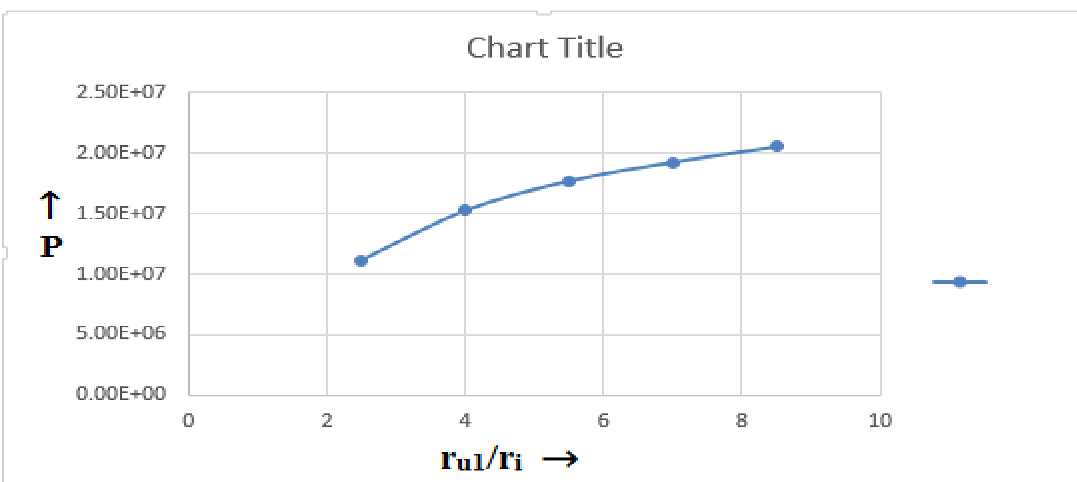


Table 5.12: Effect of four symmetric eccentric holes on the laminated composite annular plates (AS-3501)

In both cases, the buckling load increases with increase in distance of holes from the centre of the plate showing that stiffness of the plates can be increased by positioning holes away from the centre of the plate.

CHAPTER 6

CONCLUSIONS

1. As the ratio of r_i/r_o increases, the buckling load increases.
2. The buckling load in case of F-C boundary condition is higher than in case of C-F boundary conditions. The composite circular plate, free at the outside edge and inside edge fixed, becomes weaker, with the application of in-plane loads. This characteristic becomes more noticeable with the increment of r_i/r_o ratio.
3. The circular plate with $(90/0)_{2s}$ lamination scheme shows lower buckling load capacity than the other lamination schemes.
4. For all lamination schemes, it is observed that with increase in r_i/r_o ratio, the buckling load increases.
5. The non-dimensional buckling load increases with the increase in the thickness of the plate.
6. The non-dimensional buckling load increases as the number of layers of the plate increases, given the plates have constant thickness.
7. The buckling load increases with the increase in the distance between the hole centre and the center of the circular plate. As the hole comes closer to the center of circular plate, the circular plate becomes weaker because as the hole is moved closer to the center, it results in weakening of the inside part of the circular plate.
8. When the size of the hole is increased, it leads to weakening of plates resulting in a decrease in buckling load for a particular location of the hole from the centre of the plate.
9. The buckling load decreases with the increase in the number of off center holes and it increases as the off center hole moves away from the center of the plate.

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